PHASE PORTRAITS OF A CLASS OF VAN DER POL-DUFFING LIÉNARD DIFFERENTIAL EQUATIONS

MURILO R. CÂNDIDO[1], JAUME LLIBRE[1] AND CLAUDIA VALLS[2]

Abstract. We classify the phase portraits of the van der Pol-Duffing Liénard differential equations

$$x'' + f(x)x' + g(x) = 0,$$

where $g(0) = 0$, $g'(0) > 0$, g is an odd polynomial of degree 3 such that its unique real zero is the origin and $f(x)$ is an even polynomial of degree two. These differential equations of second order can be written as the following differential systems of first order

$$\dot{x} = y, \qquad \dot{y} = -x - x^3 - a(b + x^2)y,$$

with $a, b \in \mathbb{R}$. We prove that these systems have exactly one stable limit cycle if $b < 0$, and no limit cycles when $b \geq 0$.

1. Introduction and statement of the main results

In this paper we consider differential equations of the form

$$x'' + f(x)x' + g(x) = 0, \tag{1}$$

where f and g are polynomials with $\deg(f) = n$ and $\deg(g) = m$. Here the prime denotes derivative with respect to the time t. These equations are known as generalized Liénard differential equations. They have many applications in several branches of the sciences such as chemical reactions, predator-prey type systems, vibration analysis, etc, see for instance [5, 9, 12] and the references therein.

Note that the differential equation (1) of second order is equivalent to the planar differential system of first order

$$x' = y, \qquad x' = -g(x) - f(x)y. \tag{2}$$

2010 *Mathematics Subject Classification.* Primary 34A05. Secondary 34C05, 37C10.

Key words and phrases. limit cycles, van der Pol-Duffing oscillator, Poincaré compactification.