PHASE PORTRAITS OF THE RAYLEIGHT-DUFFING OSCILLATOR WITH POSITIVE LINEAR STIFFNESS AND LINEAR DAMPING

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Abstract. In this paper we characterize the global dynamics in the Poincaré disc of the Rayleigh-Duffing oscillator with linear damping and positive linear stiffness. In particular we show that the existence of a limit cycle depends on the sign of the linear damping.

1. Introduction and statement of the main results

Rayleigh in 1875 published his book *The Theory of Sound* [7], where he shown the first physical phenomenon modeled by a limit cycle. Later on Van der Pol [8] in 1934 provided the second relevant limit cycle studying the motion of an electrical circuit in a vacuum tube. Both models have been studied intensively because they are two essential oscillators in the nonlinear dynamical systems, see for instance [5, 6].

In this paper we analyze the global phase portraits of the RayleighDuffing oscillator modeled by the second-order differential equation (3) of [4]

$$\ddot{x} + ax + x^3 + \varepsilon(2b\dot{x} + \dot{x}^3) = 0,$$

where a, b, ε are parameters being ε sufficiently small. We recall that the dot is the derivative with respect to the independent variable t, that x is the displacement, a is the stiffness and b is related with the linear damping.

This second-order differential equation is equivalent to the first order differential system

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -ax - x^3 - \varepsilon(2by + y^3). \end{aligned} \tag{1}$$

Note that this system is symmetric with respect to the origin, that is, it is invariant with respect to the transformation $(x, y) \mapsto (-x, -y)$. It is also invariant under the change of variables $(x, y, \varepsilon, t) \mapsto (-x, y, -\varepsilon, -t)$, and so

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