

ACHIEVABLE CONNECTIVITIES OF FATOU COMPONENTS FOR A FAMILY OF SINGULAR PERTURBATIONS

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ABSTRACT. In this paper we study the connectivity of Fatou components for maps in a large family of singular perturbations. We prove that, for some parameters inside the family, the dynamical planes for the corresponding maps present Fatou components of arbitrarily large connectivity and we determine precisely these connectivities. In particular, these results extend the ones obtained in [Can17, Can18].

Keywords: holomorphic dynamics, Fatou and Julia sets, singular perturbation, connectivity of Fatou components.

1. INTRODUCTION

In the recent decades there has been an increasing interest in studying families of rational maps usually called *singular perturbations*. Roughly speaking, a family is called a singular perturbation if it is defined by a *base* family (called the *unperturbed family* and for which we have a deep understanding of the dynamical plane) plus a *local* perturbation, that is, a perturbation which has a significant effect on the orbits of points in some part(s) of the dynamical plane, but a very small dynamical relevancy on other regions.

Singular perturbations, no matter the concrete formulas, have some common properties which make their study interesting. On the one hand, the degree of the unperturbed family is smaller than the degree of the perturbed one. Consequently, one should expect richer dynamics for singular perturbations than for the unperturbed maps. On the other hand, most of this new freedom arising from the perturbation may be captive of the dynamical properties of the unperturbed family. The balance between these two scenarios has become very successful in finding new dynamical phenomena.

The relation between the topology of the dynamically invariant sets (Fatou and Julia set) and the behaviour of the critical orbit(s) is an important issue when studying the dynamical plane of a particular rational map. A paradigmatic example of this is the Dichotomy Theorem for the quadratic family. In this way, singular perturbations are somehow a perfect scenario to observe new phenomena for the invariant sets with respect to the unperturbed maps, for which we usually observe a tame topology. Indeed, the main goal of this paper is to investigate in this direction and to prove that for a certain family of singular perturbations we can construct examples for which, in the same dynamical plane, there are Fatou components with given arbitrarily high connectivities.

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