POLYNOMIAL AND RATIONAL FIRST INTEGRALS FOR NON–AUTONOMOUS POLYNOMIAL HAMILTONIAN SYSTEMS

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ABSTRACT. Known results on the existence of polynomial and rational first integrals for autonomous polynomial Hamiltonian systems are extended to non–autonomous polynomial Hamiltonian systems invariant under an involution.

The key tool for proving these results is the existence of Darboux polynomials for the non–autonomous polynomial Hamiltonian systems.

1. Introduction and statement of the main results

In this work we deal with the non–autonomous polynomial Hamiltonian systems

$$\dot{q}_i = \frac{\partial H(q,p,t)}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H(q,p,t)}{\partial q_i}, \quad i = 1, \ldots, m, \tag{1}$$

with Hamiltonian $H(q,p,t)$, where $q = (q_1, \ldots, q_m) \in \mathbb{C}^m$ and $p = (p_1, \ldots, p_m) \in \mathbb{C}^m$ are the generalized coordinates. The dot denotes derivative with respect to the time t. Usually q is called the *position vector* and p the *momenta vector*.

In this work we extend the results on the autonomous polynomial Hamiltonian differential systems obtained in [9] to the non–autonomous polynomial Hamiltonian systems (1). More precisely, using the existence of some involution under which the Hamiltonian system (1) remains invariant and the existence of Darboux polynomials, we provide sufficient conditions in order that the non–autonomous polynomial Hamiltonian systems (1) have a second polynomial or rational first integral independent of the Hamiltonian first integral $H(q,p,t)$. These first integrals are polynomial or rational in the variables q and p with coefficients C^1 functions in the time $t \in \mathbb{R}$.

We denote by X_H the associated Hamiltonian vector field in $\mathbb{C}^{2m}$ to the Hamiltonian system (1), i.e.,

$$X_H = \sum_{i=1}^{m} \frac{\partial H(q,p,t)}{\partial p_i} \frac{\partial}{\partial q_i} - \sum_{i=1}^{m} \frac{\partial H(q,p,t)}{\partial q_i} \frac{\partial}{\partial p_i} + \frac{\partial}{\partial t}.$$

Let U be an open subset of $\mathbb{C}^{2m}$. Then a non–locally constant function $I : U \to \mathbb{C}$ such that it is constant on the orbits of the Hamiltonian vector field X_H contained in U is called a *first integral* of X_H in U, i.e. $X_H I \equiv 0$ on U. Our first integrals depend on time, some authors called the first time-dependent integrals as invariants. See for example, [2], [4], [6], [11], [13].

2010 *Mathematics Subject Classification.* 37J35, 37K10.

Key words and phrases. Non–autonomous Polynomial Hamiltonian systems; polynomial first integrals; rational first integrals; Darboux polynomial.