

PHASE PORTRAITS OF (2;0) REVERSIBLE VECTOR FIELDS WITH SYMMETRICAL SINGULARITIES

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ABSTRACT. In this paper we study the phase portraits in the Poincaré disk of the reversible vector fields of type (2;0) having generic bifurcations around a symmetric singular point p . We also prove the nonexistence of any periodic orbit surrounding p .

1. INTRODUCTION AND MAIN RESULTS

Given two real C^k , $k \geq 1$, functions of two variables $P, Q : \mathbb{R}^2 \rightarrow \mathbb{R}$ we define a *planar C^k differential system* as a system of the form

$$(1) \quad \dot{x} = P(x, y), \quad \dot{y} = Q(x, y),$$

where the dot in system (1) denotes the derivative with respect to the independent variable t . We call the map $X = (P, Q)$ a *vector field*. If P and Q are polynomials then system (1) is a *planar polynomial differential system*. In this case we say that system (1) has *degree n* if the maximum of the degrees of P and Q is n . If $n = 1$ then system (1) is called a *linear differential system*. This last class of systems is already completely understood (see for instance chapter 1 of [17]). However for $n \geq 2$, that is for nonlinear differential polynomial systems we know very few things. The class of planar polynomial systems with degree $n \geq 2$ is too wide, so is common to study more specific subclasses and to classify their topological phase portraits. See for instance [1], [10] and [23].

In this paper we are concerned with the *reversible vector fields*. Given a C^k vector field X (not necessarily planar) and a C^k diffeomorphism $\varphi : \mathbb{R}^m \rightarrow \mathbb{R}^m$ satisfying $\varphi^2 = id_{\mathbb{R}^m}$ we say that X is φ -reversible of type $(m; r)$, for $r \in \{0, 1, \dots, m\}$, if

$$D\varphi(z)X(z) = -X(\varphi(z))$$

for all $z \in \mathbb{R}^m$ and $Fix(\varphi) = \{z \in \mathbb{R}^m : \varphi(z) = z\}$ is a r -dimensional manifold. Many types of reversible vector fields have been studied for several authors. For example in [22] all the low codimension singularities of systems

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