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Final evolutions of a class of May-Leonard Lotka-Volterra systems

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We study a particular class of Lotka-Volterra 3-dimensional systems called May-Leonard systems, which depend on two real parameters a and b, when $a+b=-1$. For these values of the parameters we shall describe its global dynamics in the compactification of the non-negative octant of $\mathbb{R}^3$ including its infinity. This can be done because this differential system possesses a Darboux invariant.

Keywords: May-Leonard system; Lotka-Volterra system; invariant, global dynamics.

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1. Introduction

Polynomial ordinary differential systems are often used in various branches of applied mathematics, physics, chemist, engineering, etc. Models studying the interaction between species of predator-prey type have been extensively analyzed as the classical Lotka-Volterra systems. For more information on the Lotka-Volterra systems see for instance [8] and the references quoted there. In particular, one of these competition models between three species inside the class of 3-dimensional Lotka-Volterra systems is the *May-Leonard model* given by the polynomial differential system in $\mathbb{R}^3$

$$\begin{aligned}\dot{x} &= x(1-x-ay-bz),\\ \dot{y} &= y(1-bx-y-az),\\ \dot{z} &= z(1-ax-by-z),\end{aligned} \tag{1.1}$$

where a and b are real parameters and the dot denotes derivative with respect to the time t. See for more details on the May-Leonard system the papers [10] and [2] and on Lotka-Volterra systems [9], and the references quoted there.

The Lotka-Volterra systems in $\mathbb{R}^3$ have the property that the three coordinate planes are invariant by the flow of these systems. Moreover, at points of straight line $x=y=z$, system (1.1) is reduced to $\dot{x}=x-(1+a+b)x^2$, because the other equations do not provide any further information. Therefore, the bisectrix of the non-negative octant is an invariant straight line for this differential system.

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