DYNAMICAL SYSTEMS
2020, VOL. 35, NO. 2, 361–368
https://doi.org/10.1080/14689367.2019.1703906

On the dynamics of the Euler equations on so(4)

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ABSTRACT
This paper deals with the Euler equations on the Lie Algebra so(4). These equations are given by a polynomial differential system in $\mathbb{R}^6$. We prove that this differential system has four 3-dimensional invariant manifolds and we give a complete description of its dynamics on these invariant manifolds. In particular, each of these invariant manifolds are fulfilled by periodic orbits except in a zero Lebesgue measure set.

ARTICLE HISTORY
Received 8 February 2019
Accepted 6 December 2019

KEYWORDS
Euler equations on so(4); periodic orbits; reversible systems

1. Introduction

The Euler equations on the Lie algebra so(4) have a very long story that can be found in [2, 4, 5, 8, 10–12] and in the references quoted therein. The Euler differential equations on so(4) in $\mathbb{R}^6$ depend on six parameters λ_i, $i = 1, \ldots, 6$, and are given by

$$\begin{aligned}
\dot{x_1} &= (\lambda_3 - \lambda_2)x_2x_3 + (\lambda_6 - \lambda_5)x_6x_5,\\
\dot{x_2} &= (\lambda_1 - \lambda_3)x_1x_3 + (\lambda_4 - \lambda_6)x_4x_6,\\
\dot{x_3} &= (\lambda_2 - \lambda_1)x_1x_2 + (\lambda_5 - \lambda_4)x_4x_5,\\
\dot{x_4} &= (\lambda_3 - \lambda_5)x_3x_5 + (\lambda_6 - \lambda_2)x_2x_6,\\
\dot{x_5} &= (\lambda_4 - \lambda_3)x_3x_4 + (\lambda_1 - \lambda_6)x_1x_6,\\
\dot{x_6} &= (\lambda_2 - \lambda_4)x_2x_4 + (\lambda_5 - \lambda_1)x_1x_5,
\end{aligned} \tag{1}$$

where the dot denotes derivative with respect to the time t.

Equation (1) can have a very complicated dynamics, due to the complexity of the system: nonlinear, high dimension and many parameters. From the integrability point of view, in the paper [7] it is proved that the Euler equations on the Lie algebra so(4) with a diagonal quadratic Hamiltonian either satisfy the Manakov condition, or have at most four functionally independent polynomial first integrals.

In the present paper, our first interest was about the existence or non-existence of periodic orbits for system (1). As we will see in what follows, we have proved that there exist three-dimensional invariant manifolds for the flow of system (1) that are fulfilled by periodic orbits.

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