# LIMIT CYCLES FOR SOME FAMILIES OF SMOOTH AND NON-SMOOTH PLANAR SYSTEMS

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Abstract. In this paper, we apply the averaging method via Brouwer degree in a class of planar systems given by a linear center perturbed by a sum of continuous homogeneous vector fields, to study lower bounds for their number of limit cycles. Our results can be applied to models where the smoothness is lost on the set $\Sigma = \{xy = 0\}$. We also apply them to present a variant of Hilbert 16th problem, where the goal is to bound the number of limit cycles in terms of the number of monomials of a family of polynomial vector fields, instead of doing this in terms of their degrees.

## 1. Introduction

A limit cycle is a periodic orbit isolated in the set of all periodic orbits in a differential system. The existence of limit cycles became important in the applications to the real world, because many phenomena are related with their existence, see for instance the Van der Pol oscillator [27, 28]. One of the useful tools to detect such objects is the averaging theory. We refer to the book of Sanders and Verhulst [25] and to the book of Verhulst [29] for an introduction of this subject. Buica and Llibre in [5], generalized the averaging theory for studying periodic solutions of continuous differential systems using mainly the Brouwer degree.

The theory of piecewise smooth differential system has been developing very fast and it has become certainly an important common frontier between Mathematics, Physics and Engineering for example. In many works on piecewise smooth differential system the set $\Sigma$, where the systems lose smoothness, is a regular manifold. But a few years ago it was increasing the study of the case where $\Sigma$ can be the union of regular manifolds, which includes, the case when $\Sigma$ is not regular, but it is an algebraic manifold. See for instance Panazzolo and Da Silva in [21]. Also there are works that deal with the search of limit cycles of discontinuous systems with $\Sigma$ being an algebraic manifold, see for instance [16] and [19].

In this work we give some lower bounds for the number of limit cycles in some classes of continuous, non necessarily locally Lipschitz, piecewise smooth differential systems with $\Sigma = \{xy = 0\}$. The main technique will be the averaging theory via Brouwer degree developed in [5, 6].

In Section 2 we explain some of the problems that have motivated our study. They include systems that model the capillary rise, some population models and also some type of SIR models. All of them have in common that can be written as differential equations of the form

$$\dot{x} = f(x, y, \sqrt{x}, \sqrt{y}), \quad \dot{y} = g(x, y, \sqrt{x}, \sqrt{y}), \tag{1}$$

with $f$ and $g$ smooth or polynomial functions. Extending the function $\sqrt{u}$ as $\operatorname{sgn}(u)\sqrt{|u|}$ these systems can be considered in the full plane but they are non-smooth on the set

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