ZERO-HOPF BIFURCATION IN A MALARIA MODEL

REBIHA BENTERKI[1] AND JAUME LLIBRE[2]

ABSTRACT. This paper is devoted to study the zero–Hopf bifurcations of a Malaria dynamic model with transgenic mosquitoes. More precisely, we provide the values of the parameters for which this model exhibits simultaneously two zero–Hopf equilibria, and we provide sufficient conditions in order that from both equilibria a periodic orbit bifurcates. Few differential systems exhibit simultaneuous two zero-Hopf bifurcations.

1. INTRODUCTION

Malaria is one of the most mortifying infection in the world which is caused by mosquitoes. Mathematical models have been used to give an adequate study to understand the transmission of Malaria in human population for ever 100 years, , see [1]. Many researchers studied the role of the transgenic mosquitoes in order to reduce the transmission of the Malaria. One of the most and remarkable models is the one where the authors examined the possibility to replacing wild mosquitoes by transgenic ones, in which they established a model of Malaria transmission, and by using Floquet theory [3] they studied the existence and stability of the disease–free equilibrium points.

Based on Kermack and Mckendrick assumptions of the model given in [9] and the epidemic model given in [2, 8], Liu et al. studied the behaviors and the numerical simulations of Malaria dynamic models with transgenic mosquitoes, such model depends on eight parameters. These authors provided the conditions for which the equilibrium points of these models are asymptotically stable.

First they considered the model

$$\begin{aligned} \dot{x} &= \beta(1-x)y - \gamma x, \\ \dot{y} &= \alpha x(1-a-y)-z) - \mu y - cay, \end{aligned} \tag{1}$$

at a fixed proportion a with $0 \leq a < 1$. After they studied this model at a changeable proportion, i.e.

$$\begin{aligned} \dot{x} &= \beta(1-x)y - \gamma x, \\ \dot{y} &= \alpha x(1-y-z) - \mu y - cyz, \\ \dot{z} &= \delta_1 yz + \delta_2 z(1-y-z) - \omega z, \end{aligned} \tag{2}$$

where α, β, γ, μ, ω, a, c, δ_1 and δ_2 are positive constants. The description of these parameters in systems (1) and (2) is shown in Table 1.

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