THE SOLUTION OF THE SECOND PART OF THE 16TH HILBERT PROBLEM FOR NINE FAMILIES OF DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS

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ABSTRACT. We provide the exact maximum number of limit cycles of some families of discontinuous piecewise differential systems formed by two differential systems separated by a straight line, when these differential systems are linear centers or three families of cubic isochronous centers. These maximum number of limit cycles vary from 0, 1, 2, 3, 5 and 7 depending of the chosen families.

1. Introduction and the statement of the main results

We consider a polynomial differential systems in $\mathbb{R}^2$ of the form

$$\frac{dx}{dt} = P(x,y), \qquad \frac{dy}{dt} = Q(x,y), \tag{1}$$

where the degree of the systems is the maximum degree of the polynomials P and Q.

In 1900 David Hilbert [15] gave a talk at the International Congress of Mathematicians in Paris, where he provided a list of 23 problems. One of these problems which remain open up to now is the second part of the 16-th problem, in which Hilbert asked for an upper bound for the maximum number of limit cycles of all polynomial differential systems of a given degree, see also [19, 20].

In this paper we consider the discontinuous piecewise differential systems

$$X^{\pm} : (\dot{x}, \dot{y}) = (f^{\pm}(x,y), g^{\pm}(x,y)), \tag{2}$$

defined in the half-planes $\Sigma^{\pm} = \{(x,y) \in \mathbb{R}^2 : \pm x > 0\}$. On the straight line $\Sigma = \{x = 0\}$ the differential system is bivaluated. The straight line Σ is called the *straight line of discontinuity* when the two vector fields $X^{\pm}$ do not coincide on it. We use the Filippov conventions for defining the discontinuous piecewise differential system on Σ, see [9]. If $f^+(0,y)f^-(0,y) > 0$ at the point $(0,y) \in \Sigma$ we say that $(0,y)$ is a *crossing point*. If a periodic orbit of a discontinuous piecewise differential system (2) has exactly two crossing points we say that it is a *crossing periodic orbit*.

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