THE LIMIT CYCLES OF PIECEWISE LINEAR DIFFERENTIAL SYSTEMS FORMED BY CENTERS AND SEPARATED BY IRREDUCIBLE CUBIC CURVES

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Abstract. In the qualitative theory of the planar discontinuous piecewise linear differential systems one of the main problems is the study of the number of crossing limit cycles that these systems can have. We study the number of crossing limit cycles of discontinuous piecewise linear differential systems formed by centers and separated by an irreducible algebraic cubic curve. We prove that these differential systems only can exhibit 0, 1, 2 or 3 crossing limit cycles having two intersection points with the cubic of separation.

1. Introduction

One of the first works studying the discontinuous piecewise linear differential systems in the plane is due to Andronov, Vitt and Khaikin in [1]. Later on these systems became a topic of great interest in the mathematical community due to their applications for modeling real phenomena, see for instance the books [3, 19] and references there quoted.

To determine the non–existence, the existence of limit cycles and their number is one of the big problems in the qualitative theory of the planar differential systems, and in particular of the planar discontinuous piecewise linear differential systems separated by a curve Σ. In this work we are considering that a *crossing limit cycle* is a periodic orbit isolated in the set of all periodic orbits of the system which has exactly two points on the discontinuity curve Σ.

The problem of finding the best upper bound for the maximum number of limit cycles that a family of piecewise linear differential systems in the plane separated by a straight line can have, has been studied by many authors recently, see for instance [2, 5, 7, 18]. Lum and Chua[16, 17] in 1990 conjectured that the continuous (but non–smooth) piecewise linear systems in the plane separated by one straight line have at most one limit cycle. This conjecture was proved by Freire et al [6] in 1998, for a shorter proof see [11]. Han and Zhang [8] in 2010 conjectured that discontinuous piecewise linear differential systems in the plane separated by a straight line have at most two crossing limit cycles. Huan and Yang [9] in 2012 provided a negative answer to this conjecture exhibiting a numerical example with three crossing limit cycles. Llibre and Ponce in [12] proved the existence of these three limit cycles analytically. Nowadays it remains as an open problem to know

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