LIMIT CYCLES OF PLANAR DISCONTINUOUS PIECEWISE LINEAR HAMILTONIAN SYSTEMS WITHOUT EQUILIBRIA SEPARATED BY REDUCIBLE CUBICS

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Abstract. Due to their applications to many physical phenomena during these last decades the interest for studying the discontinuous piecewise differential systems has increased strongly. The limit cycles play a main role in the study of any planar differential system, but to determine the maximum number of limits cycles that a class of planar differential systems can have is one of the main problems in the qualitative theory of the planar differential systems. Thus in general to provide a sharp upper bound for the number of crossing limit cycles that a given class of piecewise linear differential system can have is a very difficult problem. In this paper we characterize the existence and the number of limit cycles for the piecewise linear differential systems formed by linear Hamiltonian systems without equilibria and separated by a reducible cubic curve, formed either by an ellipse and a straight line, or by a parabola and a straight line parallel to the tangent at the vertex of the parabola. Hence we have solved the extended 16th Hilbert problem to this class of piecewise differential systems.

1. Introduction and statement of the main results

Andronov, Vitt and Khaikin [1] started around 1920's the study of the piecewise differential systems mainly motivated for their applications to some mechanical systems, and nowadays these systems still continue to receive the attention of many researchers. Thus these differential systems are widely used to model processes appearing in mechanics, electronics, economy, etc., see for instance the books [6] and [28], and the survey [25], as well as the hundreds of references cited there.

A *limit cycle* is a periodic orbit of the differential system isolated in the set of all periodic orbits of the system. Limit cycles are important in the study of the differential systems. Thus limit cycles have played and are playing a main role for explaining physical phenomena, see for instance the limit cycle of van der Pol equation [26, 27], or the one of the Belousov-Zhavotinskii model [2, 29], etc.

The *extended* 16*th Hilbert problem*, that is, to find an upper bound for the maximum number of limit cycles that a given class of differential systems can exhibit, is in general an unsolved problem. Only for very few classes of differential system this problem has been solved. For the class of discontinuous piecewise differential systems here studied, we can obtain its solution by using the first integrals provided by the Hamiltonians of the systems which form the discontinuous piecewise differential systems. For the statement of the classical 16th Hilbert problem see [14, 16, 20].

Of course in order that a discontinuous piecewise differential system be defined on the discontinuous line, which separates the different differential systems forming the discontinuous piecewise differential system, we follow the rules of Filippov, see [9].

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