Singular values and non-repelling cycles for entire transcendental maps

Anna Miriam Benini *and Núria Fagella †
Dept. de Matemàtica y Informàtica, Universitat de Barcelona
Barcelona Graduate School of Mathematics (BGSMath)
Gran Via 585 08007, Barcelona

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Abstract

Let f be a map with bounded set of singular values for which periodic dynamic rays exist and land. We prove that each non-repelling cycle is associated to a singular orbit which cannot accumulate on any other non-repelling cycle. When f has finitely many singular values this implies a refinement of the Fatou-Shishikura inequality. Our approach is combinatorial in the spirit of the approach used by [Kiw00], [BCL$^+$16] for polynomials.

1 Introduction

Consider the iteration of an entire transcendental map $f : \mathbb{C} \to \mathbb{C}$. The map f fails to be a covering due to the presence of *singular values*, that is the set $S(f)$ of points near which not all inverse branches of f^{-1} are well defined and univalent. While the singular values of rational maps are always *critical values* (images of zeros of f' or *critical points*), transcendental functions may have also *asymptotic values*, and we have that

$$S(f) = \overline{\{\text{critical and asymptotic values for } f\}}.$$

Recall that $s \in \mathbb{C}$ is an asymptotic value if there exists a curve $\gamma : [0, \infty) \to \mathbb{C}$ such that $|\gamma(t)| \to \infty$ as $t \to \infty$ and $f(\gamma(t)) \to s$ as $t \to \infty$ (for example, $s = 0$ is an asymptotic value for the map $z \mapsto \exp(z)$, and the curve γ can be taken to be the negative real axis).

Special classes of maps are singled out in terms of their set of singular values and will be important for our discusion. More precisely define

$$\mathcal{S} = \{f : \mathbb{C} \to \mathbb{C} \text{ entire} \mid \#S(f) < \infty\}$$

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