Classifying simply connected wandering domains

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Dedicated to Misha Lyubich on his 60th birthday

Abstract

While the dynamics of transcendental entire functions in periodic Fatou components and in multiply connected wandering domains are well understood, the dynamics in simply connected wandering domains have so far eluded classification. We give a detailed classification of the dynamics in such wandering domains in terms of the hyperbolic distances between iterates and also in terms of the behaviour of orbits in relation to the boundaries of the wandering domains. In establishing these classifications, we obtain new results of wider interest concerning non-autonomous forward dynamical systems of holomorphic self maps of the unit disk. We also develop a new general technique for constructing examples of bounded, simply connected wandering domains with prescribed internal dynamics, and a criterion to ensure that the resulting boundaries are Jordan curves. Using this technique, based on approximation theory, we show that all of the nine possible types of simply connected wandering domain resulting from our classifications are indeed realizable.

1 Introduction

We consider dynamical systems defined by the iteration of holomorphic maps $f : \mathbb{C} \to \mathbb{C}$ on the complex plane, and particularly transcendental ones, that is, those with an essential singularity at infinity. The complex plane, seen as the phase space of the system, splits into two completely invariant subsets: the *Fatou set*, or those points in a neighbourhood of which the iterates $\{f^n\}$ form a normal family, and its complement, the *Julia set*. The Fatou set is open and consists typically of infinitely many connected components called *Fatou components*.

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