THE LIMIT CYCLES OF DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS FORMED BY CENTERS AND SEPARATED BY IRREDUCIBLE CUBIC CURVES II

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Abstract. In this paper we provide a lower bound for the maximum number of crossing limit cycles of some class of planar discontinuous piecewise linear differential systems formed by centers and separated by an irreducible algebraic cubic curve. First we prove that the systems constituted by three zones can exhibit 0, 1, 2, 3 or 4 crossing limit cycles having four intersection points with the cubic of separation. Second we prove that the systems constituted by two zones can exhibit 0, 1, or 2 crossing limit cycles having four intersection points with the cubic of separation.

1. Introduction

1.1. **Classification of the irreducible cubic polynomials.** A *cubic curve* is the set of points $(x, y) \in \mathbb{R}^2$ satisfying $P(x, y) = 0$ for some polynomial $P(x, y)$ of degree three. This cubic is *irreducible* (respectively *reducible*) if the polynomial $P(x, y)$ is irreducible (respectively reducible) in the ring of all real polynomials in the variables x and y.

A point (x_0, y_0) of a cubic $P(x, y) = 0$ is *singular* if $P_x(x_0, y_0) = 0$ and $P_y(x_0, y_0) = 0$. A cubic curve is *singular* if it has some singular point, as usual here P_x and P_y denote the partial derivaties of P with respect to the variables x and y respectively.

A *flex* of an algebraic curve C is a point p of C such that C is nonsingular at p and the tangent at p intersects C at least three times. The next theorem characterizes all the irreducible cubic algebraic curves.

Theorem 1. *The following statements classify all the irreducible cubic algebraic curves.*

(a) *A cubic curve is nonsingular and irreducible if and only if it can be transformed with affine transformations into one of the following two curves;*

$$c_1(x, y) = y^2 - x(x^2 + bx + 1) = 0 \quad \text{with } b \in (-2, 2), \text{ or}$$
$$c_2(x, y) = y^2 - x(x - 1)(x - r) = 0 \quad \text{with } r > 1.$$

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