DYNAMICS OF TWO EINSTEIN-FRIEDMANN COSMOLOGICAL MODELS

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Abstract. We describe completely the dynamics of the two Einstein-Friedmann cosmological models, which can be characterized by the Hamiltonians

$$H = \frac{1}{2}(p_y^2 - p_x^2) + e^{2x}V(y),$$

whith the cosmological potentials $V(y) = e^{\lambda y}$, or $V(y) = (a + by)e^y$ with $\lambda ab \neq 0$.

1. Introduction and statement of the main results

The present work is devoted to the Einstein-Friedmann cosmological models, which can be characterized by the Hamiltonian

$$H = \frac{1}{2}(p_y^2 - p_x^2) + e^{2x}V(y), \tag{1}$$

where $V(y) = e^{\lambda y}$ or $V(y) = (a + by)e^y$ with $\lambda ab \neq 0$ are cosmological potentials. For more details on these two special models see subsections 2.2 and 3.1 [10], and for more details on the general Einstein-Friedmann cosmological models see [4, 7].

The Hamiltonian system with two degrees of freedom associated to the Hamiltonian

$$H = \frac{1}{2}(p_y^2 - p_x^2) + e^{2x+\lambda y} \tag{2}$$

is

$$\begin{aligned} \dot{x} &= \frac{\partial H}{\partial p_x} = -p_x, \\ \dot{y} &= \frac{\partial H}{\partial p_y} = p_y, \\ \dot{p_x} &= -\frac{\partial H}{\partial x} = -2e^{2x+y\lambda}, \\ \dot{p_y} &= -\frac{\partial H}{\partial y} = -\lambda e^{2x+y\lambda}. \end{aligned} \tag{3}$$

The above Hamiltonian H has the additional first integral

$$F = 2p_y - \lambda p_x, \tag{4}$$

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