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DISCRETE DYNAMICAL SYSTEMS

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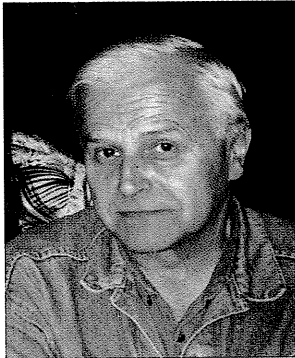
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FOREWORD



Professor Wiesław Szlenk

This is a special issue that grew out of the lectures that took place during the *Symposium on Discrete Dynamical Systems in honor of Wiesław Szlenk*, which was held in Bellaterra, Barcelona, Spain, from August 25–28, 1997.

The aim of the symposium was to honor the memory of Wiesław Szlenk who died in Barcelona after a long illness on July 26, 1995 while holding a visiting position at the Universitat Autònoma de Barcelona. At the same time, we wanted to reexperience what had been our usual activities with him: discussing mathematics, making conjectures and proving theorems. The fact that his main contributions in mathematics were in the field of Discrete Dynamical Systems motivated the topic of the symposium.

We all remember Wiesław Szlenk as a distinguished scholar, and equally as a warm and energetic colleague and friend. What can be said about him is well summarized in the following obituary that was distributed by e-mail:

Göttingen 28.07.1995

With deep regret we report the death after a long illness of

Wiesław Szlenk

in Barcelona on July 26, 1995.

Wiesław Szlenk was Professor of Mathematics at Warsaw University and an elected member of the University Senate in the current term. He held long-term visiting positions in Mexico and Barcelona, and taught at the Agricultural University in Warsaw.

He was a central figure in the formation of the dynamical systems research group at Warsaw University in the late 1960's. He played a leading role in its development into a world center during the following decade and continued to inspire several generations as advisor and mentor.

His best-known papers include *On invariant measures of expanding differentiable mappings* (with K. Krzyżewski, 1969) and *Entropy of piecewise monotone mappings* (with M. Misiurewicz, 1980).

He was also actively interested in the applications of dynamical systems, particularly to mathematical biology, and in bringing mathematics to a broader audience. He wrote two books: *An Introduction to the Theory of Smooth Dynamical Systems* and a textbook on probability theory which is used by high-school students throughout Poland.

He was for many years involved in organizing the Mathematical Olympiads for high-school students in Poland, as well as of numerous scientific meetings in Warsaw, including the International Congress of Mathematicians in 1983.

We remember Wiesław as a man of great personal generosity and incredible energy.

Wiesław Szlenk will be fondly remembered and sorely missed by his colleagues and friends in the mathematical community throughout the world.

Lluís Alsedà (Barcelona)
Stefan Jackowski (Warsaw)
Zbigniew Nitecki (Medford, Mass.)
Mariusz Urbański (Denton, Texas)

The interest of the symposium was focused on systems in one-dimensional spaces (intervals, circles, trees and graphs), as well as higher dimensional ones (tori, spheres, discs and other manifolds). Summarizing progress made in this field and exploring new directions formed an important part of the symposium. Discussions were encouraged to give people an opportunity to exchange ideas.

The symposium brought together 77 researchers from 50 different universities, and the following lectures were given:

- F. Balibrea:** *Some new results on sequence topological entropy*
J. Bobok: *Two inverses problem*
N. Fagella: *Dynamics on Hubbard trees*
J. M. Gambaudo: *Asymptotic Braid Theory*
J. Guaschi: *Surface homeomorphisms and Reidemeister classes*
M. Jakobson: *Nonhyperbolic attractors in dimensions 1 and 2*
X. Jarque: *Misiurewicz points for complex exponentials*
A. Katok: *Entropy and growth of periodic orbits for low-dimensional systems*
G. Keller: *Equilibrium states for S-unimodal maps*
S. Kolyada: *Topological entropy of nonautonomous piecewise monotone dynamical systems on the interval*
K. Krzyżewski: *On some Sidon sequences*
J. Llibre: *Sets of periods for torus maps*
J. Los: *Dynamical CW-complexes*
A. Manning: *Symbolic dynamics for a hyperbolic toral automorphism*
M. Misiurewicz: *Conditions for stability of interval maps*
Z. Nitecki: *Automorphisms of graphs and surfaces*
T. Nowicki: *Invariant measures for S-unimodal maps*
A. Nunes: *Minimum number of fixed points for maps of the figure eight space*
F. Przytycki: *Holomorphic Collet–Eckmann maps: rigidity*
C. Simó: *Invariant curves of perturbed nontwist area preserving maps*
M. Smorodinsky: *Normal \mathbb{Z}^d configurations*
J. Smítal: *On the entropy $\log 2$ problems*
S. van Strien: *Update about fibonacci maps*
D. Treschev: *Closures of separatrices in two-dimensional area-preserving maps*
Ch. Tresser: *The dynamics of some digital filters*
M. Urbański: *Apollonian Packings*
P. Walters: *Strong equilibrium states*
M. P. Wojtkowski: *Hamiltonian systems with linear potential and collisions*
X. Ye: *Topological entropy of a tree map with non-divisible periodic orbits*
P. Zgliczyński: *Sharkovskii theorem for multidimensional perturbations of 1-dim maps*

All participants were invited to submit a contribution to these proceedings, and all papers that were accepted had to pass through a refereeing process appropriate for a mathematical research journal.

We thank the following institutions for the economic support they provided: *Centre de Recerca Matemàtica, Ministerio de Educación y Cultura, Universitat Autònoma de Barcelona* and *Grupo de Sistemas Dinámicos de la Universidad de Murcia*. We also thank the participants of the meeting for offering everyone an intense and highly interesting mathematical experience. Finally, we thank Maria Juliá for the invaluable help in preparing this issue.

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