TOPOLOGICAL ENTROPY, SETS OF PERIODS AND TRANSITIVITY FOR GRAPH MAPS

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ABSTRACT. Transitivity, the existence of periodic points and positive topological entropy can be used to characterize complexity in dynamical systems. It is known that for graphs that are not trees, for every $\varepsilon > 0$, there exist (complicate) totally transitive maps (then with cofinite set of periods) such that the topological entropy is smaller than ε (simplicity). First we will show by means of three examples that for any graph that is not a tree the relatively simple maps (with small entropy) which are totally transitive (and hence robustly complicate) can be constructed so that the set of periods is also relatively simple. To numerically measure the complexity of the set of periods we introduce a notion of a *boundary of cofiniteness*. Larger boundary of cofiniteness means simpler set of periods. With the help of the notion of boundary of cofiniteness we can state precisely what do we mean by extending the entropy simplicity result to the set of periods: *there exist relatively simple maps such that the boundary of cofiniteness is arbitrarily large (simplicity) which are totally transitive (and hence robustly complicate)*. Moreover, we will show that, the above statement holds for arbitrary continuous degree one circle maps.

1. Introduction

Transitivity, the existence of infinitely many periods and positive topological entropy often characterize the complexity in dynamical systems. This paper aims at showing that totally transitive maps on graphs, despite of being complicate in the above sense can have somewhat simple sets of periods (simplicity with respect to topological entropy was already known). To be more precise and to state the main results of the paper we need to introduce some basic notation.

Let X be a topological space and let $f\colon X \longrightarrow X$ be a map. A point $x \in X$ is called a *periodic point of f of period n* if $f^n(x) = x$ and n is the minimum positive integer with this property. The set of all positive integers n such that f has a periodic point of period n is denoted by $\mathsf{Per}(f)$. A set of periods is called *cofinite* if its complement (on $\mathbb{N}$) is finite or, equivalently, it contains all positive integers larger than a given period.

Date: June 21, 2018.

1991 *Mathematics Subject Classification.* Primary: 37B40, 54H20, 37E45, 37E25.

Key words and phrases. Topological entropy, sets of periods, total transitivity, boundary of cofiniteness, rotation sets, graph maps.

Acknowledgements: The authors have been partially supported by MINECO grant numbers MTM2014-52209-C2-1-P and MTM2017-86795-C3-1-P. Lluís Alsedà acknowledges financial support from the Spanish Ministry of Economy and Competitiveness, through the "María de Maeztu" Programme for Units of Excellence in R&D (MDM-2014-0445). The second author has been partially supported by CNPq–Brasil.