AN ALGORITHM TO COMPUTE ROTATION NUMBERS IN THE CIRCLE

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Abstract. In this article we present an efficient algorithm to compute rotation intervals of circle maps of degree one. It is based on the computation of the rotation number of a monotone circle map of degree one with a constant section. The main strength of this algorithm is that it computes *exactly* the rotation interval of a natural subclass of the continuous non-invertible degree one circle maps.

We also compare our algorithm with other existing ones by plotting the Devil's Staircase of a one-parameter family of maps and the Arnold Tongues and rotation intervals of some special non-differentiable families, most of which were out of the reach of the existing algorithms that were centred around differentiable maps.

1. Introduction

The rotation interval plays an important role in combinatorial dynamics. For example Misiurewicz's Theorem [9] links the set of periods of a continuous lifting F of degree one to the set $M := \{n \in \mathbb{N} : \frac{k}{n} \in \operatorname{Rot}(F) \text{ for some integer } k\}$, where $\operatorname{Rot}(F)$ denotes the rotation interval of F. Moreover, it is natural to compute lower bounds of the topological entropy depending on the rotation interval [1]. In any case, the knowledge of the rotation interval of circle maps of degree one is of theoretical importance.

The rotation number was introduced by H. Poincaré to study the movement of celestial bodies [14], and since then has been found to model a wide variety of physical and sociological processes. The application to voting theory [8, 12] is specially surprising in this context.

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Date: December 6, 2020.

2020 *Mathematics Subject Classification.* Primary 37E10, Secondary 37E45.

Key words and phrases. Rotation number, circle maps, nondecreasing degree one lifting, algorithm.

Supported by the Spain's "Agencial Estatal de Investigación" (AEI) grants MTM2017-86795-C3-1-P and MDM-2014-0445 within the "María de Maeztu" Program.