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THIRTY YEARS AFTER SHARKOVSKIĬ'S THEOREM: NEW PERSPECTIVES

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THIRTY YEARS AFTER SHARKOVSKIĬ'S THEOREM: NEW PERSPECTIVES

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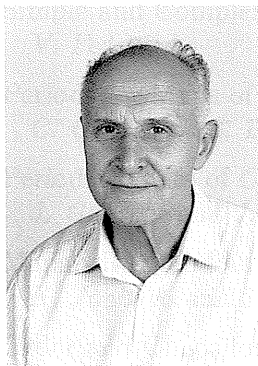
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FOREWORD

This special issue contains a collection of papers on Combinatorial Dynamics. They are an outgrowth of the lectures that took place during the international symposium, *Thirty Years after Sharkovskii's Theorem: New Perspectives*, which was held at La Manga del Mar Menor, Murcia, Spain, from June 13 to June 18, 1994.

In 1964, A. N. Sharkovskii published a paper on the coexistence of periods for interval maps (the English translation of this paper appears in these proceedings). He showed that the apparent complexity in the periodic behavior of those maps was based on a total ordering of the positive integers, so there is some order inside chaos. Since the appearance of this paper several lines of research have been developed. One of them is the analysis of the periodic structure on spaces of dimension one: intervals, circles, trees and graphs. Another is the same analysis but in spaces of dimension greater than one: tori, spheres, discs and other manifolds. These studies also revealed interesting applications to some models which help in understanding a number of phenomena from biology, economics, physics, etc. The aim of the meeting was twofold: on the one hand to summarize the progress made in this field and on the other to explore new directions.

The meeting gathered together 61 researchers and 43 lectures were given. All speakers were invited to submit a contribution to these proceedings. All papers that were received went through the refereeing process typical of a mathematical research journal and those that were accepted form the present volume.



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We would also like to thank Will Geller for his help in preparing this issue.

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