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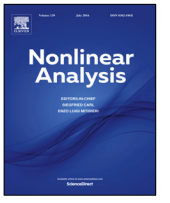

Orbitally universal centers

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ABSTRACT

In this paper we define when a polynomial differential system is orbitally universal and we show the relevance of this notion in the classical center problem, i.e. in the problem of distinguishing between a focus and a center.

1. Introduction and statement of the main results

In this work we consider differential systems in $\mathbb{R}^2$ of the form

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

with P and Q polynomials having at the origin an isolated singular point. As usual the dot denotes derivative with respect to the time t. Along this paper we also consider the associated *vector field* $\mathcal{X} = P(x, y)\partial/\partial x + Q(x, y)\partial/\partial y$ to the differential system (1).

One of the main open problem in the qualitative theory of dynamical systems is to characterize when a singular point of system (1) has a center. This problem is known as the *center problem* and it consists in distinguishing between a center and a focus. A center is a singular point for which there exists a punctured neighborhood filled of periodic orbits, and a focus has a punctured neighborhood filled of spiraling orbits. We note that the center problem goes back to Poincaré [32] and Dulac [17].

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