THE DYNAMICS OF THE RELATIVISTIC KEPLER PROBLEM

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Abstract. We deal with the Hamiltonian system (HS) provided by the correction given by the special relativity to the motion of the two–body problem, or by the first order correction to this problem coming from the general relativity. This Hamiltonian system is completely integrable with the Hamiltonian H and the angular momentum C. We have two objectives.

First, we describe the global dynamics of the Hamiltonian system (HS) in the following sense. Let I_h (respectively I_c) be the subset of the phase space where $H = h$ (respectively $C = c$). Since H and C are first integrals, the sets I_h, I_c and $I_{hc} = I_h \cap I_c$ are invariant under the flow of the Hamiltonian system (HS). We determine the global dynamics on those sets when h and c vary.

Second, recently Tudoran in [19] provided a criterion which detects when a non–degenerate equilibrium point of a completely integrable system is Lyapunov stable. Every equilibrium point q of the completely integrable Hamiltonian system (HS) is degenerate and has zero angular momentum, so the mentioned criterion cannot be applied to it. But we will show that this criterion is also satisfied when it is applied to the Hamiltonian system (HS) restricted to zero angular momentum.

1. Introduction

The Manev Hamiltonian is

$$\mathcal{H} = \frac{1}{2}\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) + \frac{a}{r} + \frac{b}{r^2},$$

where a and b are arbitrary constant. This Hamiltonian describes the motion of a two–body problem defined by the potential $a/r + b/r^2$, where r is the distance between the two particles.

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