# Translation arcs and Lyapunov stability in two dimensions

RAFAEL ORTEGA

*Departamento de Matemática Aplicada,*
*Universidad de Granada,*
*18071 Granada*
*Spain*

*E-mail address:* `rortega@ugr.es`
*URL:* `http://www.ugr.es/~ecuadif/fuentenueva.htm`

Given a closed orbit $\gamma$ of a system of differential equations in the plane

$$\dot{x} = X(x), \quad x \in \mathbb{R}^2,$$

the index of the vector field $X$ around $\gamma$ is one. This classical result has a counterpart in the theory of discrete systems in the plane. Consider the equation

$$x_{n+1} = h(x_n), \quad x_n \in \mathbb{R}^2,$$

where $h$ is an orientation-preserving homeomorphism and assume that there is a recurrent orbit that is not a fixed point. Then there exists a Jordan curve $\gamma$ such that the fixed point index of $h$ around this curve is one. The proof is based on the theory of translation arcs, initiated by Brouwer. This talk is dedicated to discuss some consequences of the above result, specially in stability theory. We will compute the indexes associated to a stable invariant object and show that Lyapunov stability implies persistence (in two dimensions). The invariant sets under consideration will be fixed points, periodic orbits and Cantor sets. The more recent results on Cantor sets are joint work with Alfonso Ruiz-Herrera.