Combination of scaling exponents and geometry of the action of renormalization operators

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Renormalization group ideas are important for describing universal properties of different routes to chaos: period-doubling in unimodal maps, quasiperiodic transitions in circle maps, dynamics on the boundaries of Siegel disks, destruction of invariant circles of area-preserving twist maps, etc. The universal scaling exponents for each route are related to the properties of the corresponding renormalization operators.

In [1, 2], we proposed the *Principle of Approximate Combination of Scaling Exponents* (PACSE) that organizes the scaling exponents for different transitions to chaos. Roughly speaking, if the combinatorics of a transition is a composition of two simpler combinatorics, then the scaling exponent of this transition is approximately equal to the product of the scaling exponents corresponding to the two simpler transitions. In [1, 2], we stated PACSE quantitatively as precise asymptotics of the scaling exponents for the combined combinatorics, and gave convincing numerical evidence for it for the four dynamical systems mentioned above.

We propose an explanation of PACSE in terms of the dynamical properties of the renormalization operators—in particular, as a consequence of certain transversal intersections of the stable and unstable manifolds of the operators corresponding to different transition to chaos. As an essential ingredient in this picture, we prove [3] a general shadowing theorem that works for infinite dimensional discrete dynamical systems that are not necessarily invertible (which is the case of the renormalization operators acting in appropriate function spaces).

References

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