Modified Lotka-Volterra maps and their interior periodic points

PETER MALIČKÝ

Department of Mathematics, Faculty of Natural Sciences, Matej Bel University, Tajovského 40, 974 01 Banská Bystrica, Slovakia

E-mail address: `peter.malicky@umb.sk`

Consider the plane triangle

$$\Delta = \{ [x,y] : 0 \le x,\ 0 \le y,\ x+y \le 4 \}$$

and the map

$$F : \Delta \to \Delta,\ [x,y] \mapsto [x(4-x-y),\ xy]\ .$$

(We denote by $[x,y]$ a point in the plane, while (α,β) and $\langle \alpha,\beta \rangle$ are open and closed intervals on the real line.) In [4] A. N. Sharkovskiĭ formulated some problems about properties of a map which is conjugated with the map F. This map was studied in [1, 2, 3, 5] and is called a Lotka-Volterra map (in [1, 2, 3]). It is easy to show that a point $P = [x,0] \in \Delta$ is a periodic point of the map F if and only if $x = 4\sin^2 \frac{k\pi}{2^n \pm 1}$, where $n \ge 1$ and k are integers with $0 \le 2k < 2^n \pm 1$. We are interested in *interior* periodic points of the map F. Our main result of [3] is a relation between lower and interior periodic points. Namely, if a point $[4\sin^2 \frac{k\pi}{2^n \pm 1}, 0]$ is a saddle point of the map F^n then there is a interior periodic point with the same itinerary with respect to the sets

$$\Delta_L = \{[x,y] : 0 \le x < 2\,, 0 \le y \le 4-x\}$$

and

$$\Delta_R = \{[x,y] : 2 < x \le 4\,, 0 \le y \le 4-x\}\,.$$

We extend this result for modifications of the map F which are defined as follows. Assume that for any $x \in (0,\ 4)$ we have an increasing homeomorphism φ_x of the interval $\langle 0,\ 4-x \rangle$ onto itself. Moreover let the function $\varphi(x,y) = \varphi_x(y)$ be continuous in the domain

$$\widehat{\Delta} = \{ [x,y] : 0 < x < 4\,, 0 \le y \le 4-x \}\ .$$

Let $G : \Delta \to \Delta$ be defined by

$$G[x,y] = \begin{cases} [0,0] & \text{if } x = 0 \text{ or } x = 4 \ , \\ [x(4-x-\varphi(x,y)), x\varphi(x,y)] & \text{otherwise.} \end{cases}$$

Then the map G is called a modified Lotka-Volterra map. Note that $F[x,0] = G[x,0]$ for all $x \in \langle 0,\ 4\rangle$. We construct two modifications G for which all lower fixed points of the map G^n are repulsive and saddle points respectively. We give an example of a modification G such that for any $n \geq 1$ all repulsive lower fixed points of the map F^n are saddle points of G^n and vice versa.

References

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