An answer to some problems on self-similar sets and the open set condition

JAKUB ŠOTOLA

Mathematical Institute in Opava,
Silesian University in Opava,
Na Rybníčku 1, 746 01, Opava,
Czech Republic

E-mail address: Jakub.Sotola@math.slu.cz

The subject of this talk are self-similar sets (a *self-similar* set E is a fixed point of map $\varphi(E) = \cup\varphi_i(E)$, where φ_i are contractive similitudes on $\mathbb{R}^n$, see [3]), especially those satisfying the so-called open set condition (OSC). These sets have many "nice" properties, but they are still raising many questions too.

The OSC requires existence of an open set G, such that

$$\varphi(G) \subset G \text{ and } \varphi_i(G) \cap \varphi_j(G) = \emptyset.$$

Such a set G is called a feasible set.

In the talk I will consider some open problems formulated by L. Feng and Z. Zhou in their papers [1] and [2]. Namely I will present a counterexample to the existence of connected (or even convex) feasible sets and i will prove that, if the self-similar set satisfies OSC, the fixed points of maps φ_i must be distinct for different indeces.

References

[1] L. Feng, Z. Zhou, *Twelve open problems on the exact value of the Hausdorff measure and on topological entropy: a brief survey of recent results*, Nonlinearity **17** (2004), 493–502.

[2] L. Feng, Z. Zhou, *Some problems on fractal geometry and topological dynamical systems*, Anal. Theor. Appl. **25** (2007), 5–15.

[3] K. J. Falconer, *The geometry of fractal sets*, Cambridge: Cambridge University Press, 1985