

On the dynamics of two exponential type systems of difference equations

G. PAPANICOLAOU¹, C. J. SCHINAS²

¹ School of Engineering, Democritus University of Thrace, Xanthi, 67100, Greece.
E-mail address: gpapas@env.duth.gr

² School of Engineering, Democritus University of Thrace, Xanthi, 67100, Greece.
E-mail address: cschinas@ee.duth.gr

In this paper we investigate the boundedness and the persistence of the positive solutions, the existence of a unique positive equilibrium and the global asymptotic stability of the equilibrium of the following systems of difference equations

$$x_{n+1} = a + by_{n-1}e^{-x_n}, \quad y_{n+1} = c + dx_{n-1}e^{-y_n}, \quad (1)$$

$$x_{n+1} = a + by_{n-1}e^{-y_n}, \quad y_{n+1} = c + dx_{n-1}e^{-x_n}, \quad (2)$$

where the constants a, b, c, d are positive real numbers and the initial values x_{-1}, x_0, y_{-1}, y_0 are also positive real numbers. We note that if $x_{-1} = y_{-1}, x_0 = y_0$ then $x_n = y_n$, for all $n = -1, 0, \dots$ and so both systems reduce to the difference equation $x_{n+1} = \alpha + \beta x_{n-1}e^{-x_n}$ which has been studied in [1]. System (2) represents the rule by which two discrete, competing populations reproduce from one generation to the next. Variables x_n , and y_n , denote population sizes during the n -th generation and the sequence or orbit (x_n, y_n) , $n = 0, 1, 2, \dots$ describes how the populations evolve over time.

References

- [1] E. El-Metwally, E.A. Grove, G. Ladas, R. Levins, M. Radin, *On the difference equation, $x_{n+1} = \alpha + \beta x_{n-1}e^{-x_n}$* , *Nonlinear Analysis* **47** (2001), 4623-4634.