

Asymptotics for second-order linear q -difference equations

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We will work on the q -uniform lattice $q^{\mathbb{N}_0} := \{q^k : k \in \mathbb{N}_0\}$ with $q > 1$ or, possibly, on $q^{\mathbb{Z}}$. We will introduce the class of functions satisfying the relation

$$y(qt)/y(t) \sim \omega(t) \quad \text{as } t \rightarrow \infty,$$

where ω is a nonzero function. We will study its properties and show how this class is related e.g. to the class of q -regularly functions or to the class of q -hypergeometric functions. Then we will consider the second-order linear q -difference equation

$$y(q^2t) + a(t)y(qt) + b(t)y(t) = 0,$$

where $b(t) \neq 0$ and $a(t)$ are real functions. Sufficient and necessary conditions will be presented for this equation to have solutions in the above mentioned class. Related results concerning estimates for solutions and (non)oscillation of all solutions will also be discussed.

References

- [1] Pavel Řehák, *On a certain asymptotic class of solutions to second-order linear q -difference equations*, J. Phys. A: Math. Theor. **45** (2012) 055202 (19pp).