On periodic solutions of 2–periodic Lyness difference equations

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We study the existence of periodic solutions of the *non–autonomous periodic Lyness' recurrence*

$$u_{n+2} = \frac{a_n + u_{n+1}}{u_n}, \qquad (1)$$

where $\{a_n\}_n$ is a cycle with positive values a,b and with positive initial conditions.

It is known that for $a = b = 1$ all the sequences generated by this recurrence are 5–periodic. Among other results concerning periodic solutions, we prove:

Proposition *Consider the* 2*–periodic Lyness' recurrence (1) for* $a > 0$*,* $b > 0$ *and positive initial conditions* u_1 *and* u_2*.*

(i) If $(a, b) \neq (1, 1)$*, then there exists a computable value* $p_0(a, b) \in \mathbb{N}$ *such that for any* $p > p_0(a, b)$ *there exist continua of initial conditions giving rise to* $2p$*–periodic sequences.*

(ii) The set of prime periods arising when $(a, b) \in (0, \infty)^2$ *and positive initial conditions are considered, contains all the even numbers except* 4, 6, 8, 12 *and* 20*. If* $a \neq b$*, then it does not appear any odd period, except* 1*.*

References

[1] G. Bastien, V. Mañosa, M. Rogalski. *On periodic solutions of 2-periodic Lyness difference equations*. arXiv:1201.1027v1 [math.DS]