

# Efficient synchronization of one-dimensional chaotic quadratic maps coupled without symmetry

ROSÁRIO LAUREANO<sup>1</sup>, DIANA A. MENDES<sup>2</sup>,  
MANUEL A. MARTINS FERREIRA<sup>3</sup>

<sup>1</sup> *Department of Quantitative Methods, IBS – ISCTE IUL Business School, Lisbon, Portugal.*

<sup>2</sup> *Department of Quantitative Methods, IBS – ISCTE IUL Business School, Lisbon, Portugal.*

<sup>3</sup> *Department of Quantitative Methods, IBS – ISCTE IUL Business School, Lisbon, Portugal.*

We consider synchronization phenomena of chaotic discrete dynamical systems with unidirectional and bidirectional coupling mechanisms. It is studied a non-linear coupling scheme that appears in natural a family of analytic complex quadratic maps (Isaeva *et al.* [1]). It is an asymmetric coupling between two real quadratic maps in which we use different values of the control parameters chosen in the region of chaos. The map obtained by coupling two chaotic quadratic maps exhibits a richer dynamics than the single one, but it is still possible to study its behaviour. We are not aware about any studies of this type of coupling. When practical synchronization (in the Kapitaniak sense) is not achieved, but the difference between the dynamical variables of the systems is bounded, we still can apply to the coupled system a chaos control technique based on the well-known OGY-method [2] (Ott-Grebogoy-Yorke), the pole-placement control technique, developed by Romeiras *et al.* [3], in order to decrease the difference between the dynamical variables. Moreover, we obtain stable identical and generalized synchronization with some versions of the original coupling, highlighting the absence of symmetry. Two of them are generalizations promoting the use of different parameters coupling. By analyzing the difference between the dynamical variables of the systems, we obtain some results leading to stable synchronization. In case of coupling with two different coupling parameters, the linear stability of the synchronous state is ensured when some relations are guaranteed between the coupling parameters and the initial conditions.

## References

- [1] O.B. Isaeva, S.P. Kuznetsov and V.I. Ponomarenko (2001), Mandelbrot set in coupled logistic maps and in an electronic experiment, *Phys. Rev. E* **64**, R055201-4.
- [2] E. Ott, C. Grebogy and J.A. Yorke (1990), Controlling chaos, *Phys. Rev. Lett.* **64** (11), 1196-1199.
- [3] F.J. Romeiras, C. Grebogi, O.E. Ott and W.P. Dayawansa (1992), Controlling chaotic dynamical systems, *Phys. D* **58**, 165-192