On some stochastic competition models

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In his recent review, Schreiber [1] describes the state of the art for stochastic competition models of the general form

$$X^i_{t+1} = f_i(X_t, \xi_t)X^i_t, \;\; i = 1, \dots k, \;\; t = 0, 1, 2, \dots$$

where the state space **S** is a subset of $\Re^k_+$ and the union of the coordinate axes in $\Re^k_+$ forms the *extinction set* $\mathbf{S}_0$; the ξ's represent a randomly evolving environment.

I will look at some variants and special cases of this model. Specifically, we will limit ourselves to two populations, $k = 2$, and the functions f_i are chosen to be of the *Ricker* type:

$$\exp(r^i_t - K^i_t(X^i_t + \alpha^j X^j_t)), \;\; i, j = 1, 2, \;\; i \neq j.$$

Here the r^i_t's model the average intrinsic per capita rate of growth of population i at time t. The growth is attenuated by the negative term where K^i_t describes the intra-specific competition at time t and α^j (assumed constant over time) the relative importance of the inter-specific competition.

Another variant of the model is concerned with *demographic* stochasticity. Here our aim is to study the evolution of two finite populations as size-dependent branching processes, which on average follow the above Ricker-type model, with non-random r_i, K_i. We want to describe the long-term behavior and compare it with that of the corresponing deterministic model. The size-dependent branching processes will necessarily have finite life-times. Can anything be said about these?

References

[1] Sebastian J. Schreiber, *Persistence for stochastic difference equations: A minireview,* , Journal of Difference Equations and Applications (2012) (to appear). Available via the author's web site `www-eve.ucdavis.edu/sschreiber/pubs.shtml`