W-maps and harmonic averages

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The W-map [1] is a transformation $\tau : [0,1] \to [0,1]$ with a graph in the shape of letter W. We assume that it is continuous, piecewise linear with four branches: τ_1, τ_2, τ_3 and τ_4. The τ_1 and τ_3 are decreasing, the τ_2 and τ_4 increasing. The first and the last branches are onto, while the middle branches meet at point $(1/2, 1/2)$, i.e., $1/2$ is a turning fixed point of τ. The modulus of the slope of τ_i is $s_i > 1$, $i = 1, 2, 3, 4$, so τ is piecewise expanding and as such admits an absolutely continuous invariant measure (acim) μ. Let us consider a family of small perturbations τ_n of map τ, such that $\tau_n \to \tau$ as $n \to \infty$. Let τ_n have acim μ_n. If $\mu_n \to \mu$, we call τ "acim-stable". It was shown in [1] that W-map with $s_2 = s_3 = 2$ is not acim-stable. It turns out that acim-stability of τ depends on $1/s_2 + 1/s_3$ which is related to harmonic average of s_2 and s_3 equal $\frac{2}{1/s_2+1/s_3}$. If $1/s_2 + 1/s_3 < 1$, then τ is acim-stable. We prove this slightly improving the classical Lasota-Yorke inequality [2]. If $1/s_2 + 1/s_3 > 1$, then we can produce a family of τ_n's such that $\mu_n \to \delta_{\{1/2\}}$ weakly. If $1/s_2 + 1/s_3 = 1$, then we can produce a family of τ_n's exact on $[0, 1]$ such that μ_n's converge weakly to a combination of μ and $\delta_{\{1/2\}}$.

References

[1] Keller, G., *Stochastic stability in some chaotic dynamical systems*, *Monatshefte für Mathematik* **94** (4) (1982) 313–333.

[2] Eslami, P. and Góra, P., *Stronger Lasota-Yorke inequality for piecewise monotonic transformations*, preprint, available at http://www.peymaneslami.com/EslamiGora_Stronger_LY_Ineq.pdf