Global bifurcation analysis and applications of a Liénard polynomial system

VALERY A. GAIKO

United Institute of Informatics Problems, National Academy of Sciences of Belarus, Leonid Beda Str. 6-4, Minsk 220040, Belarus

E-mail address: valery.gaiko@gmail.com

We consider Liénard polynomial systems. There are many examples in the natural sciences and technology in which Liénard systems are applied. They are often used to model either mechanical or electrical, or biomedical systems, and in the literature, many systems are transformed into Liénard type to aid in the investigations. They can be used, e. g., in certain mechanical systems, when modeling wind rock phenomena and surge in jet engines. Such systems can be also used to model resistor-inductor-capacitor circuits with non-linear circuit elements. Recently a Liénard system has been shown to describe the operation of an optoelectronics circuit that uses a resonant tunnelling diode to drive a laser diode to make an optoelectronic voltage controlled oscillator. There are also some examples of using Liénard type systems in ecology and epidemiology.

In this talk, we discuss the general Liénard polynomial system with an arbitrary (but finite) number of singular points in the form

$$\dot{x} = y, \quad \dot{y} = -x\,(1 + \beta_1\,x + \ldots + \beta_{2l}\,x^{2l}) + y\,(\alpha_0 + \alpha_1\,x + \ldots + \alpha_{2k}\,x^{2k}). \tag{1}$$

Applying a canonical system with field rotation parameters,

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x\,(1 + \beta_1\,x \pm\, x^2 + \ldots + \beta_{2l-1}\,x^{2l-1} \pm\, x^{2l}) \\ &\quad + y\,(\alpha_0 + x + \alpha_2\,x^2 + \ldots + x^{2k-1} + \alpha_{2k}\,x^{2k}), \end{aligned} \tag{2}$$

where $\beta_1,\ \beta_3, \ldots,\ \beta_{2l-1}$ are fixed and $\alpha_0,\ \alpha_2, \ldots,\ \alpha_{2k}$ are field rotation parameters, and using geometric properties of the spirals filling the interior and exterior domains of limit cycles, we carry out the global bifurcation analysis of (2) and prove the following theorem.

Theorem. *The general Liénard polynomial system (1) can have at most $k+l$ limit cycles, k surrounding the origin and l surrounding one by one the other its singularities.*