

On the Second Order Quadratic Rational Difference

$$\text{Equation } x_{n+1} = \frac{\alpha}{(1+x_n)x_{n-1}}$$

INESE BULA

*Department of Mathematics of University of Latvia and
Institute of Mathematics and Computer Science of University of Latvia
Rīga,
Latvia*

E-mail address: ibula@lanet.lv

We investigate the periodic nature of solutions of a rational difference equation

$$x_{n+1} = \frac{\alpha}{(1+x_n)x_{n-1}}. \quad (1)$$

Classically rational difference equations are explored with nonnegative parameters and nonnegative initial conditions. We show that the rational difference equation (1) with negative initial conditions or/and with negative parameter α have different behaviour from equations with nonnegative parameters and with nonnegative initial conditions.

We explore [1, Open Problem 3.3] that requires to determine all periodic solutions of equation (1). We can assert that, for example, for difference equation (1) with parameter $\alpha > 0$ does not exist initial conditions $x_{-1} > 0$ and $x_0 > 0$ such that solution of equation (1) is periodic with prime period 5 but if $\alpha < 0$, then exist initial conditions $x_{-1} = x_0 > 0$ such that solution of equation (1) is periodic with prime period 5 (for example, $\alpha = -\frac{4}{3}$ and $x_{-1} = x_0 = 1$). Period 7 is first period for which exists nonnegative parameter α and nonnegative initial conditions (for example, $\alpha \approx 1, 053218$ and $x_{-1} = 5, x_0 = 2$).

References

- [1] A.M. Amleh, E. Camouzis, G. Ladas, *On the Dynamics of a Rational Difference Equations, Part 1*, International Journal of Difference Equations **3** (2008), 1-35.

*This work was partially supported by the Latvian Council of Science research project 09.1220.