

P-recursive moment sequences of piecewise D-finite functions and Prony-type algebraic systems

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Algebraic Signal Sampling, an active area of research in Signal Processing, aims at reconstructing an unknown finite-parametric model from a finite number of measurements such as power moments or Fourier coefficients ([2, 3, 4, 6, 7] and references therein). In many cases under consideration, the measurement sequence is P-recursive ([11]), so that the reconstruction process essentially amounts to recovery of the unknown coefficients of the corresponding recurrence relation. Important practical questions such as conditions for unique reconstruction and accuracy (stability) of solutions are directly expressible in the language of these recurrences, their perturbations and the corresponding algebraic systems [1] (so-called "Prony-type" - [8, 9]).

Continuing our previous work [2] on moment inversion for piecewise D-finite ([10]) functions, we have recently obtained a non-trivial upper bound on the number of measurements needed for unique reconstruction in terms of the ODE itself in some "singular" cases. The particular structure of the corresponding finite difference operator played a major role in the proof.

In addition to the above result and related questions, we will also discuss reconstruction of certain 2D domains and the corresponding recurrences [5].

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