

# Initial Condition Problems for Second Order Rational Difference Equations

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Often initial conditions of difference equations have a great influence on existence and behavior of solution. We investigate the initial value effect on the behavior of solutions of second order rational difference equations, for example, in [1, Open Problem 2.9.4]. is as follows: It is known that all solutions of difference equations

$$x_{n+1} = \frac{1 + x_{n-1}}{1 + x_n} \quad \text{and} \quad x_{n+1} = 1 + \frac{x_{n-1}}{x_n}, \quad n = 0, 1, 2, \dots \quad (1)$$

converges to a solution with period two:  $\dots, \phi, \psi, \phi, \psi, \dots$ . Determine  $\phi$  and  $\psi$  in terms of the initial conditions  $x_{-1}$  and  $x_0$ . Although the given equations seem very simple, however it is very complicated even to calculate numerically some terms of the solution sequence (for example, if we take first equation and initial values  $x_{-1} = x_0 = 2$  we get a solution sequence:  $2; 2; 1; \frac{3}{2}; \frac{4}{5}; \frac{25}{18}; \frac{162}{215}; \frac{9245}{6786}; \dots$ ).

Other problems connected with initial values are problems where the set of all initial points  $(x_{-1}, x_0) \in R \times R$  through which the given equation is well defined for all  $n \geq 0$  has to be gained.

On the basis of obtained results and computational experiments some ideas about initial value problems will be discussed.

## References

- [1] M.R.S. Kulenovic, G. Ladas, *Dynamics of Second Order Rational Difference Equations. With Open Problems and Conjectures.*, Chapman and Hall/CRC., USA, 2002.

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