Chaoticity and invariant measures for two population models

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First, we present a short introduction to an ergodic approach to chaotic systems based on [1]. The main idea is to show that a semiflow possesses an invariant mixing measure positive on open sets. From this it follows that the system is topological mixing and its trajectories are turbulent in the sense of Bass. Then we present two cell population models which lead to chaotic systems. The first one [2] describes the evolution of maturity of blood cells in the bone marrow and is given by a partial differential equation. The equation generates a semiflow acting on densities (i.e. integrable functions with the integral one). Next, we consider a classical structured model of cells reproduction system [3] given by a partial differential equations with a non-local division term. This equation generates semiflows acting on some subspaces of locally integrable functions. We show that our semiflows are isomorphic to the shift semiflow $f(x) = f(x + t)$ on properly chosen spaces of functions Y. The second step is to construct a mixing and invariant measure m supported on the space Y. We can do it, if we find a Gaussian process with trajectories from the space Y. Then the measure m of a Borel subset A of Y is the probability that trajectories of the process are from the set A. It should be noted that most of the recent papers concerning chaos for semigroups of operators are based on studying spectral properties of their infinitesimal generators. This approach seems to be easier than ours. But, in our opinion, the approach based on the isomorphism with shift semigroups and using invariant measures reveals why our semiflows are chaotic. The second advantage of the ergodic theory approach is that we can prove much stronger results concerning chaos.

References

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