Distributional chaos for operators with full scrambled sets

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In recent years many researchers were looking for conditions that yield complex, nontrivial dynamics of linear operators (note that, to admit such behaviour, the space must be infinite dimensional). Probably the most studied is the notion of hypercyclicity, that is, the existence of vectors $x \in X$ such that the orbit of this vector $x, T(x), T^2(x), \dots$ under the action of a continuous and linear operator $T\colon X \to X$ on a topological vector space (most often Banach or Fréchet space) X forms a dense subset of X. Distributional chaos was introduced by Schweizer and Smital as a natural extension of Li-Yorke chaos and we consider its strongest notion of *uniform distributional chaos*, which requires the existence of an uncountable set $D \subset X$ and $\varepsilon > 0$ such that for every $t > 0$ and every distinct $x, y \in D$ the upper densities of the sets $\{i \in \mathbb{N};\ \|T^i x - T^i y\| \geq \varepsilon\}$ and $\{i \in \mathbb{N};\ \|T^i x - T^i y\| < t\}$ are equal to 1. The set D is called a *distributionally ε-scrambled set*.

We answer in the negative the question of whether hypercyclicity is sufficient for distributional chaos for a continuous linear operator (we even prove that the mixing property does not suffice). Moreover, we show that a extremal situation is possible: There are (hypercyclic and non-hypercyclic) operators such that the whole space consists, except zero, of distributionally irregular vectors.