Chaotic solution for the Black-Scholes equation

HASSAN EMAMIRAD[1], GISÈLE RUIZ GOLDSTEIN[2],
JEROME A. GOLDSTEIN[3]

[1] *School of Mathematics, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5746, Tehran, Iran*
Laboratoire de Mathématiques, Université de Poitiers. teleport 2, BP 179, 86960 Chasseneuil du Poitou, Cedex, France.
E-mail address: emamirad@ipm.ir, emamirad@math.univ-poitiers.fr

[2] *Department of Mathematical Sciences, The University of Memphis, Memphis, TN 38152, USA.*
E-mail address: ggoldste@memphis.edu

[3] *Department of Mathematical Sciences, The University of Memphis, Memphis, TN 38152, USA.*
E-mail address: jgoldste@memphis.edu

The Black-Scholes semigroup is studied on spaces of continuous functions on $[0,\infty)$ which may grow at both 0 and at ∞, which is important since the standard initial value is an unbounded function. We prove that in the Banach spaces

$$Y^{s,\tau} := \{u \in C((0,\infty)) \ : \ \lim_{x\to\infty} \frac{u(x)}{1+x^s} = \lim_{x\to 0} \frac{u(x)}{1+x^{-\tau}} = 0\}$$

with norm $\|u\|_{Y^{s,\tau}} = \sup_{x>0} \left| \frac{u(x)}{(1+x^s)(1+x^{-\tau})} \right| < \infty$, the Black-Scholes semigroup is strongly continuous and chaotic for $s > 1, \tau \geq 0$ with $s\nu > 1$, where $\sqrt{2\nu}$ is the volatility. The proof relies on the Godefroy-Shapiro hypercyclicity criterion.