Graph theoretic structure of maps of the Cantor space

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We develop unifying graph theoretic techniques to study the dynamics and the structure of the spaces $H(X)$ and $C(X)$, the space of homeomorphisms and the space of continuous self-maps of the Cantor space X, respectively. Using our methods, we give characterizations which determine when two homeomorphisms of the Cantor space are conjugate to each other. We also give a new characterization of the comeager conjugacy class of the space $H(X)$. The existence of this class was established by Kechris and Rosendal in [9] and a specific element of this class was described concretely by Akin, Glasner and Weiss in [1]. Our characterization readily implies many old and new dynamical properties of elements of this class. For example, we show that no element of this class has a Li-Yorke pair, implying the well known Glasner-Weiss result [8] that there is a comeager subset of $H(X)$ each element of which has topological entropy zero. Our analogous investigation in $C(X)$ yields a surprising result: there is a comeager subset of $C(X)$ such that any two elements of this set are conjugate to each other by an element of $H(X)$. Our description of this class also yields many old and new results concerning dynamics of a comeager subset of $C(X)$.

References

[1] E. Akin, E. Glasner and B. Weiss, *Generically there is but one self homeomorphism of the Cantor set*, Trans. Amer. Math. Soc. **360** (2008), no. 7, 3613–3630.

[2] E. Akin, M. Hurley and J. Kennedy, *Dynamics of topologically generic homeomorphisms*, Mem. Amer. Math. Soc. **164** (2003), no. 783.

[3] F. Blanchard, E. Glasner, S. Kolyada and A. Maass, *On Li-Yorke pairs*, J. Reine Angew. Math. **547** (2002), 51–68.

[4] L. Block and J. Keesling, *A characterization of adding machine maps*, Topology Appl. **140** (2004), no. 2-3, 151–161.

[5] J. Buescu and I. Stewart, *Lyapunov stability and adding machines*, Ergodic Theory Dynam. Systems **15** (1995), 271–290.

[6] E. D'Aniello and U. B. Darji, *Chaos among self-maps of the Cantor space*, J. Math. Anal. Appl. **381** (2011), no. 2, 781–788.

[7] E. D'Aniello, U. B. Darji and T. H. Steele, *Ubiquity of odometers in topological dynamical systems*, Topology Appl. **156** (2008), no. 2, 240–245.

[8] E. Glasner and B. Weiss, *The topological Rohlin property and topological entropy*, Amer. J. Math. **123** (2001), no. 6, 1055–1070.

[9] A. S. Kechris and C. Rosendal, *Turbulence, amalgamation, and generic automorphisms of homogeneous structures*, Proc. London Math. Soc. (3) **94** (2007), no. 2, 302–350.

[10] M. Mazur, *Weak shadowing for discrete dynamical systems on nonsmooth manifolds*, J. Math. Anal. Appl. **281** (2003), no. 2, 657–662.

[11] J. K. Truss, *Generic automorphisms of homogeneous structures*, Proc. London Math. Soc. (3) **65** (1992), no. 1, 121–141.