Some results on the size of escaping sets

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Let f be a transcendental entire function in the class B, that is, the set of singular values of f is bounded. We present some new results about the Hausdorff dimension and Hausdorff measure of the escaping set $I(f)$ and various subsets of it.

We show that for any given sequence (p_n) tending to ∞, the set of escaping points with $|f^n(z)| \leq p_n$ always has Hausdorff dimension at least 1, and that there are functions f for which this set can be 'larger' than the fast escaping set of f (in a certain sense). This result contrasts with the situation for exponential maps, since in this case it is known that the fast escaping set has a larger Hausdorff dimension than the set points that escape slowly.

Further, we set $B_\rho := \{f \in B : f \text{ has order } \rho\}$. We show that the set $I(f)$ has infinite Hausdorff measure with respect to a certain gauge function h_ρ for every $\rho \geq 1/2$ and $f \in B_\rho$. On the other hand, for $\tilde{\rho}$ large enough, we prove the existence of a function $f \in B_{\tilde{\rho}}$ such that $I(f)$ has zero measure with respect to h_ρ. This means that the escaping sets of functions in class B of finite order can become smaller as the order increases.