On the existence of absorbing domains in Baker domains

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Let U be a hyperbolic domain in $\mathbb{C}$ and let $f : U \to U$ be a holomorphic map. An invariant domain $W \subset U$ is called *absorbing in* U *for* f if for every compact set $K \subset U$ there exists $n = n(K) > 0$, such that $f^n(K) \subset W$. The problem of existence of absorbing domains for a given f and U has a long history, since such sets are useful in many problems in dynamics.

Based in the Denjoy-Wolf Theorem (on dynamics of holomorphic maps on the unit disc), Cowen proved the existence of a simply connected absorbing domain $V \subset \mathbb{H}$ for holomorphic maps $G : \mathbb{H} \to \mathbb{H}$ (where $\mathbb{H}$ denotes the right half plane) such that $G^n \to \infty$ as $n \to \infty$. He also showed the existence of a semiconjugacy (actually a conjugacy on V) of the map G to a Möbius transformation T acting on Ω where $\Omega \in \{\mathbb{H}, \mathbb{C}\}$.

Later, König used Cowen's Theorem to prove the existence of simply connected absorbing domains in Baker domains of meromorphic maps with finitely many poles. Moreover König also showed, by means of an example, that simply connected absorbing domains do not always exist.

The main result we present here is the existence of (possibly multiply connected) absorbing domains in the general case (putting especial attention on the case of Baker domains for transcendental meromorphic functions). In another talk, Xavier Jarque will show how to use this result to prove the connectedness of the Julia set for transcendental meromorphic maps having no weakly repelling fixed points.