Conformal dimension and combinatorial modulus: applications to rational maps

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A fundamental quasisymmetry numerical invariant of a compact metric space X is its conformal dimension $\dim_{AR} X$. It was introduced by P. Pansu in order to classify, up to quasi-isometry, homogeneous spaces of negative curvature [3, 2]. Motivated by Sullivan's dictionary [4, 1], which establishes a fundamental correspondence between the properties of hyperbolic groups and of a particular class of finite branched coverings, I will define this invariant in the context of rational maps. I will show how to compute $\dim_{AR} X$ using the critical exponent Q_M associated to the combinatorial modulus, which is a discrete version of the conformal modulus from complex analysis. Finally, I will apply this result to compute the conformal dimension of the Julia sets of some hyperbolic rational maps.

References

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