Turan-type inequalities and Taylor domination for solutions of linear ODE's

DMITRY BATENKOV[1], YOSEF YOMDIN[2]

[1] *Department of Matematics, Weizmann Institute of Science, Rehovot 76100, Israel.*
E-mail address: dima.batenkov@weizmann.ac.il
URL: http://www.wisdom.weizmann.ac.il/~dmitryb

[2] *Department of Matematics, Weizmann Institute of Science, Rehovot 76100, Israel.*
E-mail address: yosef.yomdin@weizmann.ac.il
URL: http://www.wisdom.weizmann.ac.il/~yomdin

Let a family of analytic functions $f_\lambda(z) = \sum_{k=0}^\infty a_k(\lambda) z^k$ be given, and let $R(\lambda)$ be the radius of convergence of f_λ. The family f_λ possesses a property of an (N, C)-uniform Taylor domination if

$$|a_k(\lambda)| R^k(\lambda) \le C \max_{i=0,\dots,N} |a_i(\lambda)| R^i(\lambda), k = N, N+1, \dots, \qquad (1)$$

with N and C not depending on λ. Taylor domination provides, in particular, a uniform bound on the number of zeroes of f_λ in each disk strictly contained in the disk of convergence.

An important example is the family R^d_λ of all rational functions of degree d. Here uniform Taylor domination follows from the classical Turan lemma ([3]). Equivalently, Taylor domination holds for solutions of a linear recurrence relation with constant coefficients.

In this talk we discuss uniform Taylor domination for solutions of linear differential equations with polynomial coefficients, or for linear recurrence relations with the coefficients polynomially depending on the index. This is the situation of the classical Poincaré-Perron theorem ([2]), however, we are not aware of any generalization of the Turan lemma to this case. We prove a weaker version, where C in (1) is replaced by C^k, and discuss a "dynamical" approach (as in some proofs of the Poincaré-Perron theorem - see [2, 1]) to the original case. We also consider Taylor domination for the sequences of the moments of a given algebraic function.

References

[1] J. Borcea, S. Friedland, B. Shapiro, Parametric Poincaré-Perron theorem with applications. J. Anal. Math. 113 (2011), 197–225.

[2] H. Poincare, Sur les Equations Lineaires aux Differentielles Ordinaires et aux Differences Finies. (French) Amer. J. Math. 7(3) (1885), 203–258.

[3] P. Turan, Eine neue Methode in der Analysis und deren Anwendungen. Akademiai Kiado, Budapest, 1953. 196 pp.