

Boundedness Character of solutions, Monotonic Character of solutions and Existence of Periodic Solutions of a Non-Autonomous Rational Difference Equation

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Our aim is to investigate the *boundedness character*, the *periodic character* and the *monotonic character* of the non-negative solutions of the following non-autonomous rational difference equation:

$$x_{n+1} = \frac{A_n x_{n-l}}{1 + \sum_{i=0}^k B_i x_i}, \quad n = 0, 1, \dots,$$

where $\{A_n\}_{n=0}^{\infty}$ is a periodic sequence of positive real numbers, $\sum_{i=0}^k B_i > 0$, $l = 0, 1, 2, \dots$, and $k = 1, 2, 3, \dots$. We will examine how the different periods of the sequence and the relationship of the terms of the sequence affect the longer term behavior of the solutions.

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