A variant of the Krein-Rutman theorem for Poincaré difference equations

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Let $\mathbf{x}_n$, $n \in \mathbb{N}$, be a nonvanishing solution of the Poincaré difference equation

$$\mathbf{x}_{n+1} = A_n \mathbf{x}_n, \qquad n \in \mathbb{N},$$

where A_n, $n \in \mathbb{N}$, are $k \times k$ real matrices such that the limit $A = \lim_{n\to\infty} A_n$ exists (entrywise). According to a Perron type theorem, the limit $\rho = \lim_{n\to\infty} \sqrt[n]{\|\mathbf{x}_n\|}$ exists and is equal to the modulus of one of the eigenvalues of A. In this talk, we show that if the solution belongs to a given order cone K in $\mathbb{R}^k$, then ρ is an eigenvalue of A with an eigenvector in K. In the case of constant coefficients, this result implies the finite-dimensional version of the Krein-Rutman theorem.

References

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