

Positive and oscillating solutions of discrete linear equations with delay

J. DIBLÍK¹, H. HALFAROVÁ², I. HLAVIČKOVÁ³

¹ Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Department of Mathematics, Faculty of Electrical Engineering and Communication, Brno University of Technology, Brno, Czech Republic.

E-mail address: diblik.j@fce.vutbr.cz, diblik@feec.vutbr.cz

² Department of Mathematics, Faculty of Electrical Engineering and Communication, Brno University of Technology, Brno, Czech Republic.

E-mail address: xhalfa06@stud.feec.vutbr.cz

³ Department of Mathematics, Faculty of Electrical Engineering and Communication, Brno University of Technology, Brno, Czech Republic.

E-mail address: hlavicka@feec.vutbr.cz

A linear $(k + 1)$ th-order discrete delayed equation $\Delta x(n) = -p(n)x(n - k)$ where $p(n)$ is a positive sequence is considered for $n \rightarrow \infty$. This equation is known to have a positive solution if the sequence $p(n)$ satisfies an inequality. Recently it was proved that if

$$p(n) \leq \left(\frac{k}{k+1}\right)^k \times \left[\frac{1}{k+1} + \frac{k}{8n^2} + \frac{k}{8(n \ln n)^2} + \cdots + \frac{k}{8(n \ln n \dots \ln_q n)^2} \right], \quad (1)$$

where $q \in \mathbb{N}_0$, then there exists a positive vanishing solution of the considered equation and the upper bound was found. We improve this result by finding even the lower bound for the positive solution, supposing the function $p(n)$ is bounded above and below by certain functions. As well we show that, in the case of an opposite inequality to (1) for $p(n)$, all solutions of the equation considered are oscillating for $n \rightarrow \infty$.

References

- [1] J. Bařtinec, J. Diblík, Z. Šmarda: *Existence of positive solutions of discrete linear equations with a single delay*, Journal of Difference Equations and Applications, **16**, Issue 9 (2010), 1047–1056.

- [2] J. Bařtinec, L. Berezansky, J. Diblík, Z. Šmarda: *A final result on the oscillation of solutions of the linear discrete delayed equation $\Delta x(n) = -p(n)x(n - k)$ with a positive coefficient*, *Abstract and Applied Analysis*, vol. 2011, Article ID 586328, 28 pages, 2011. doi:10.1155/2011/586328.
- [3] J. Diblík, I. Hlavičková: *Asymptotic upper and lower estimates of a class of positive solutions of a discrete linear equation with a single delay*, *Abstract and Applied Analysis*, vol. 2012, Article ID 764351, 12 pages, 2012. doi:10.1155/2012/764351