Transport dynamics. From the bicircular to the real Solar System problem

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Outline

- Motivation
- Goal & Models
- Methodology & results
- Conclusions

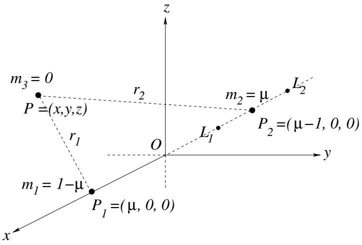
Motivation

Comets, asteroids and small particles in the solar system are capable of performing transfers from their original location to very distant places: *NATURAL TRANSPORT*

General setting

- Goal: give a dynamical mechanism to explain transport in the Solar System.
- Model for the Solar System
  - Model 1: a chain of Bicircular Problems (BCP) with the Sun, Jupiter, a different planet and a massless particle
  - Model 2: real JPL Solar system model problem and a massless particle
- Tools: Dynamical systems theory
  - unstable invariant objects and their manifolds
  - equilibrium points, periodic and quasi-periodic orbits

General framework: theoretical background and models



The **RTBP** Differential equations:

$$
\begin{aligned}
\dot{x} &= p_x + y, & \dot{p}_x &= -\partial H/\partial x,\\
\dot{y} &= p_y - x, & \dot{p}_y &= -\partial H/\partial y,\\
\dot{z} &= p_z, & \dot{p}_z &= -\partial H/\partial z,
\end{aligned}
$$

Mass ratio: $\mu = \dfrac{m_2}{m_1 + m_2}$.

Hamiltonian:

$$H(x, y, z, p_z, p_y, p_z) = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) - x p_y + y p_x - \frac{1-\mu}{r_1} - \frac{\mu}{r_2}$$

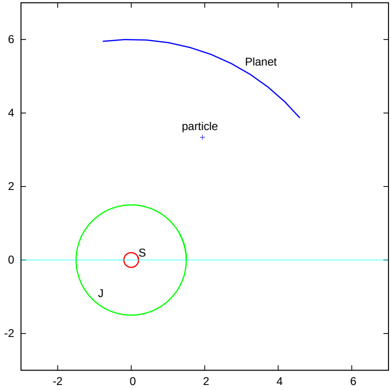
with $r_1 = \sqrt{(x-\mu)^2 + y^2 + z^2}$, $r_2 = \sqrt{(x-\mu+1)^2 + y^2 + z^2}$.
Jacobi constant:

$$C = -2H + \mu(1-\mu)$$

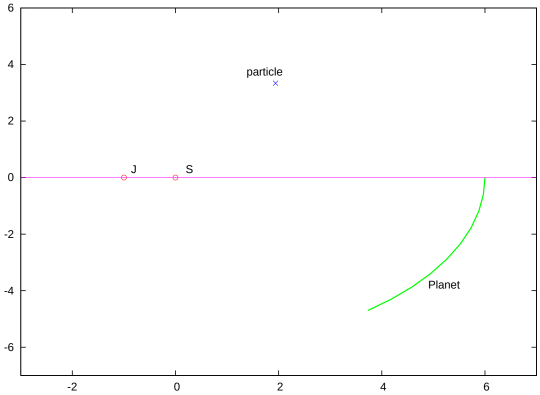
The equilibrium points L_1 and L_2 will play an important role

Model 2: the BCP

A BCP: Sun, Jupiter, Planet, a particle (S-J-Planet-particle)



Given a BCP: S-J-Planet-particle in rotating coordinates



Given a BCP: S-J-Planet-particle in rotating coordinates

Defining momenta $p_x = \dot{x} - y$, $p_y = \dot{y} + x$, $p_z = \dot{z}$, the equations may be written as a Hamiltonian system

$$\begin{aligned} H(x,y,z,p_x,p_y,p_z) &= \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) + yp_x - xp_y \\ &\quad - \frac{1-\mu}{\rho_1} - \frac{\mu}{\rho_2} - \frac{\mu_P}{\rho_P} - \frac{\mu_P}{a_P^2}(y \sin\theta - x\cos\theta) \end{aligned} \tag{1}$$

A non autonomous but periodic in t with period $T_P = \frac{2\pi}{1-n_P}$ ($\theta = \theta_0 + (1-n_P)t$).

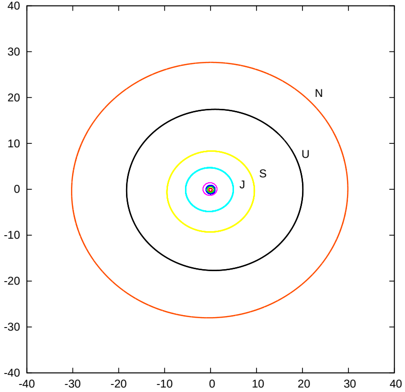
Theoretical background

- RTBP: Sun, Planet and a particle, $H_{RTBP}(u, v)$.
  - $L_{1,2}$
- BCP: Sun, Jupiter, Planet (of mass m_P) and a particle $H_{BCP}(u, v, \theta) = H_{RTBP}(u, v) + m_P * \bar{H}(u, v, \theta)$
  - $L_{1,2}$ become periodic orbits (PO), called dynamical substitutes.
  - PO is of type C*C*S

Model: The JPL Solar system

The Solar system. Transport from the exterior region to the interior one

Planets: Mercury ($ip = 1$), Venus ($ip = 2$),...., Saturn($ip = 6$), Uranus ($ip = 7$), Neptune ($ip = 8$)



Model: The JPL Solar system

- n bodies (Sun and planets are taken into account) interacting
- a particle attracted by them.
- For any given time, we take the REAL positions and velocities

Theoretical background

- RTBP: Sun, Planet and a particle, $H_{RTBP}(u, v)$.
  - $L_{1,2}$
- BCP: Sun, Jupiter, Planet (of mass m_P) and a particle $H_{BCP}(u, v, \theta) = H_{RTBP}(u, v) + m_P * \bar{H}(u, v, \theta)$
  - L_1, L_2 (PO), substitutes (C*C*S type).
- **JPL Solar System + a particle**: quasi-periodic system
  - the dynamical substitutes are quasi-periodic orbits

Papers:

- A. Jorba, C. Simó, *On quasiperiodic perturbations of elliptic equilibrium points*, SIAM J. Math. Anal. 1996
- A. Jorba, J. Villanueva, *On the persistence of lower dimensional invariant tori under quasi-periodic perturbations*, J. Nonlinear Sci. 1997

1st PART: SIMULATIONS WITH (SOME) BCP

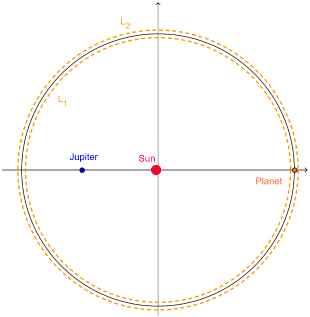
Methodology to explain transport from Neptune $ip = 8$ to Uranus $ip = 7$ (similarly from ip to $ip - 1$)

Where do we start? FIRST STEP: Computation of the dynamical substitutes.

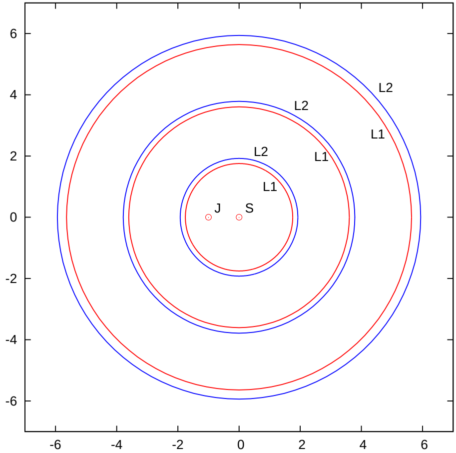
- Given a BCP: Sun-Jupiter-Planet in rotating coordinates
- L_1, L_2 (for the RTBP: Sun-Planet+particle) $\rightarrow$ computation of periodic orbits (PO) for the BCP: Sun-Jupiter-Planet+particle (multiple shooting is required)
- For each BCP, we obtain 2 unstable PO: **dynamical substitutes** of L_1 and L_2

Given a BCP: Sun-Jupiter-Planet

- 2 unstable periodic orbits, ie the dynamical substitutes of the equilibrium points L_1 and L_2 of the RTBP



Dynamical substitutes of L1, L2 for the exterior planets



Methodology to explain transport from Neptune $ip = 8$ to Uranus $ip = 7$ (similarly from ip to $ip - 1$)

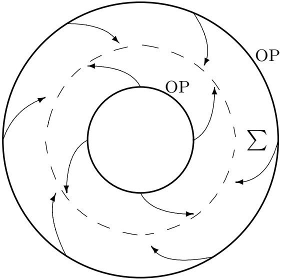
SECOND STEP

- Exterior planets: Neptune ($ip = 8$), Uranus ($ip = 7$)
- Consider a Bicircular Problem (BCP) with the Sun, Jupiter, Neptune ($ip = 8$) and a particle
- Compute the unstable manifold (inwards branch) of PO L1, $W^u(L1)$
- Consider a Bicircular Problem (BCP) with the Sun, Jupiter, Uranus ($ip = 7$) and a particle
- Compute the stable manifold (outwards branch) of PO L2, $W^s(L2)$

THIRD STEP: Look for intersections of invariant manifolds, i e heteroclinic orbits

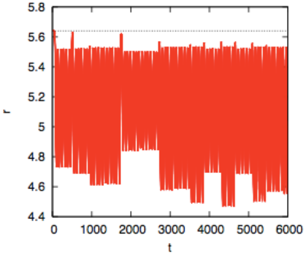
Transport using Wu and Ws

- $W^u(L1)$ and $W^s(L2)$ for $t \in [0, T_{max}]$.
- Poincaré section $\Sigma : r = ctant$
- Intersections? Heteroclinic orbits

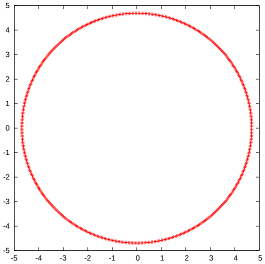


Some orbits of the manifolds for $t \in [0, T_{max}]$

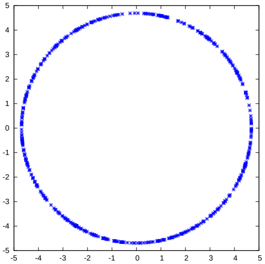
$\Sigma : r = 4.7$
Plot of $(t, r(t))$



- A given PO is parametrized by an angle θ: $\varphi(\theta) = \phi_{\frac{\theta}{2\pi} T_P}(q)$, $\phi_{T_P}(q) = q$
- Points in Σ at the FIRST crossing for $t \in [0, T_{max}]$, $T_{max} = 10000$

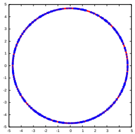


$W^u(POL1) \cap \Sigma$, (x,y)

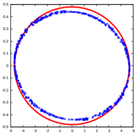


$W^s(POL2) \cap \Sigma$, (x,y)

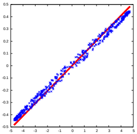
- Points in Σ at the FIRST crossing for $t \in [0, T_{max}]$



(x, y)

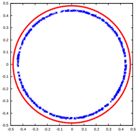


(x, p_x)



(x, p_y)

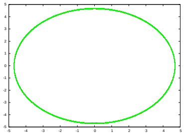
- Points in Σ at the FIRST crossing for $t \in [0, T_{max}]$, $T_{max} = 10000$



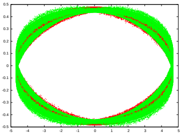
(p_x, p_y)

So "there are NO" heteroclinic connections (for this interval of time) at first crossing.

- Points in Σ, ALL the crossings for $t \in [0, T_{max}]$, $T_{max} = 50000$

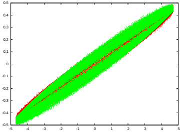


(x, y)

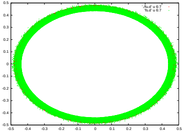


(x, p_x)

- Points in Σ, ALL the crossings for $t \in [0, T_{max}]$

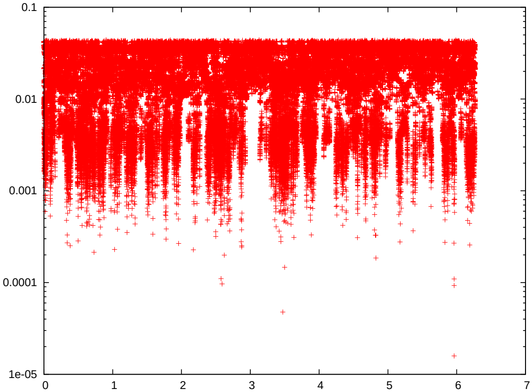


(x, p_y)



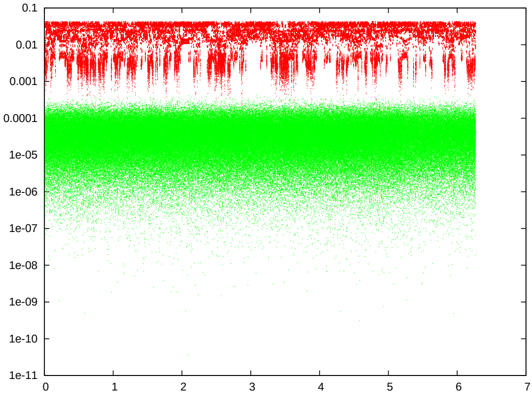
(p_x, p_y)

Minimum distance between points in Σ for the ALL the crossings for $t \in [0, T_{max}]$, $T_{max} = 50000$



$$dist_{min} \geq 0.0001$$

Minimum distance between points in Σ for the ALL the crossings for $t \in [0, T_{max}]$, $T_{max} = 50000$



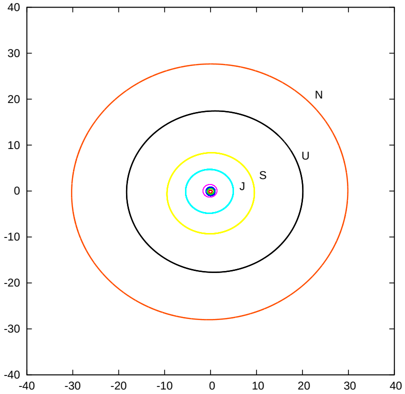
$dist_{min} \geq 0.0001, \quad dist_{min,position} = 10^{-11}$, good enough!

2nd PART: SIMULATIONS WITH THE REAL JPL SOLAR SYSTEM

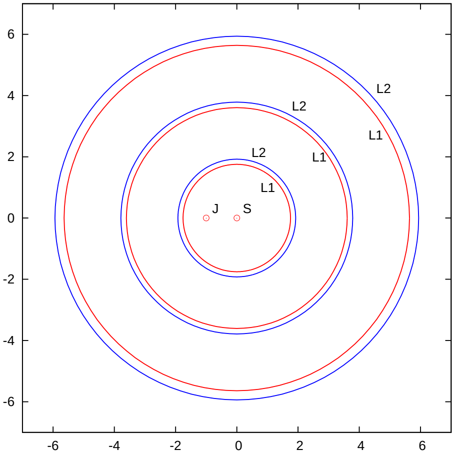
The JPL Solar system

The Solar system. 3D model. Transport from the exterior region to the interior one.

Sun and planets: Mercury, Venus,...., Saturn, Uranus, Neptune



The JPL Solar System. Transport: the same mechanism. Dynamical substitutes of L1, L2 are **quasi-periodic orbits**



Model: the JPL Solar System

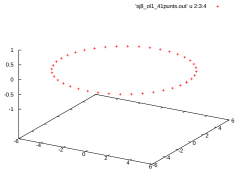
Ingredients and methodology:

- To fix ideas: let us consider the BCP Sun−Jupiter−Neptune+particle.
- The dynamical substitutes of L_1 and L_2: quasiperiodic orbits.
- We should be able to compute them and their manifolds.
- Consider the BCP Sun−Jupiter−Uranus+particle, take the associated L_1 and L_2,
- We should compute them and their manifolds.
- Check possible connections between $W^u(L1)$ (N) and $W^s(L_2)$ (U). Are there other phenomena?

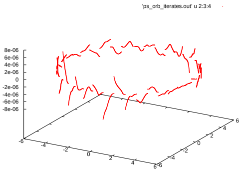
Some simulations. Computation of the dynamical substitutes:

- Representation:
  - either **in the synodical Sun-Jupiter RTBP system of coordinates**
  - or in the JPL coordinates
- Take the synodical period of Neptune in the BCP
- take n points on the PO (of the BCP) as initial seed
- Refine them using multiple shooting techniques to obtain *a quasi-periodic orbit*

Multiple shooting

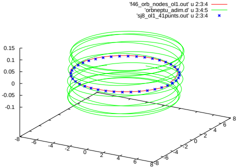


(x, y, z) (BCP)

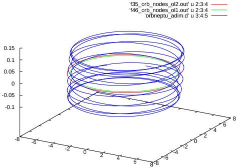


$(x, y, z)(JPL)$

Refined Dynamical substitutes and Neptune's orbit (in JPL, rotating system)

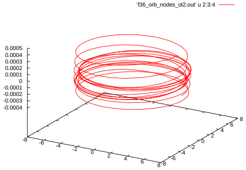


(x, y, z) (L_1)

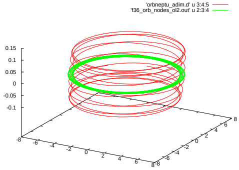


(x, y, z) (L_1, L_2)

Other DS Dynamical substitutes and Neptune's orbit (in JPL, rotating system).
We take the inertial period of Neptune.

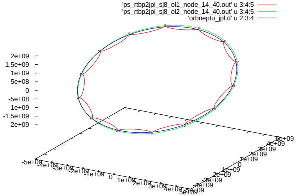


(x, y, z) (DS)



(x, y, z) (DS and Neptune's orbit)

Representation in JPL coordinates. Dynamical substitutes of L1, L2 and

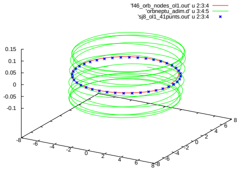


Neptune's orbit

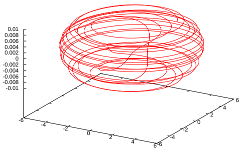
Consider the first DS L1

- We should like to see the hyperbolic behaviour...BUT
- Close encounters with other planets play a role

Consider the first DS L1 (in JPL, rotating system)
Long time span: 820 years

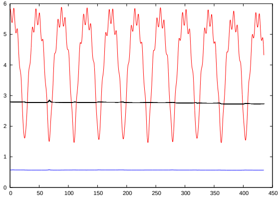


(x, y, z) (L_1)

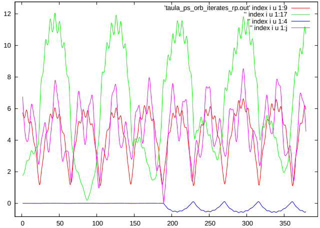


(x, y, z)

Consider the first DS L1 (in JPL, rotating system)
red: distance (particle, Sun)
black: osculating semiaxis ($a/10^9$)
blue: osculating eccentricity



red: distance (particle, Sun)
green: distance (particle, Neptune)
pink: distance (particle, Saturn)
blue: z(t) (for one of the points of the dynamical substitute)



Conclusions

- The behaviour of the manifolds of the Lyapunov periodic orbits in a chain of BCP gives a first mechanism of transport in the Solar System
- Some numerical simulations in the real Solar Sytem remain to be done...

This is a work in progress, so any suggestions are welcome....

Thank you!!