Advanced Courses in Mathematics CRM Barcelona

Jaume Llibre
Richard Moeckel
Carles Simó

Central Configurations, Periodic Orbits, and Hamiltonian Systems

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Foreword

This book collects the notes of lectures given by Jaume Llibre, Richard Moeckel, and Carles Simó at Centre de Recerca Matemàtica (CRM) in Bellaterra, Barcelona, from January 27th to 31st, 2014. The activity, in the framework of the Research Program on *Central Configurations, Periodic Orbits and Beyond in Celestial Mechanics*, hosted at CRM from January to July 2014, was a joint collaboration with the winter school in dynamical systems *Recent Trends in Nonlinear Science* (RTNS2014), promoted by the DANCE (Dinámica, Atractores y Nolinealidad: Caos y Estabilidad) Spanish network.

The Advanced Course on *Central Configurations, Periodic Orbits and Hamiltonian Systems* aimed at training their participants both theoretically and in applications in the field of nonlinear science; in this area as in many others, the theoretical and the applications points of view clearly reinforce each other.

There were three series of lectures and, accordingly, the material is distributed in three chapters in the book. The first series, delivered by Jaume Llibre, was dedicated to the study of periodic solutions of differential systems in $\mathbb{R}^n$ via Averaging Theory. Roughly speaking, in Averaging Theory one replaces a vector field by its average (over time or an angular variable) with the goal of obtaining asymptotic approximations to the original system that will be capable of guaranteeing the existence of periodic solutions. The corresponding notes in Chapter 1 start with an introduction of the classical, first order averaging theory followed by the main results of the theory for arbitrary order and dimension. The theory is applied next to the study of periodic solutions of some well known differential equations, like the van der Pol differential equation, the Liénard differential systems, or the Rossler differential system, among others. Some Hamiltonian systems are also studied.

The second series of lectures, given by Richard Moeckel, focused on methods for studying central configurations, in Chapter 2. A Central Configuration is a special arrangement of point masses interacting by Newton's law of gravitation, and with the following property: the gravitational acceleration vector produced on each mass by all the others should point toward the center of mass and be proportional to the distance to the center of mass. Central Configurations play an important role in study of the Newtonian n-body problem. For example, they lead to the only explicit solutions of the equations of motion, they govern the behavior of solutions near collisions, and they influence the topology of integral manifolds. The lectures dealt with questions about the existence and enumeration of various types of Central Configurations, including algebraic-geometrical approaches to Smale's Sixth Problem: is the number of Central Configurations always finite?

Chapter 3 is devoted to the last series of lectures, given by Carles Simó. They describe the main mechanisms leading to a fairly global description of the dynamics in conservative systems, either in the continuous version described by a Hamiltonian, or in the discrete version. The Newtonian n-body problem belongs to the general class of Hamiltonian systems. The chapter starts with several simple but paradigmatic examples in the 2D case, from which it is easier to grasp the main underlying ideas, also useful in higher dimension. Next, general theoretical results are presented and applied to different problems in Celestial Mechanics, with a rich variety of goals.

We would like to express our gratitude to the director and staff of the Centre de Recerca Matemàtica for making possible this activity. Finally, our special thanks to the three lecturers, Jaume Llibre, Richard Moeckel and Carles Simó, for the enthusiasm they showed during the course and for their fine preparation of these notes. It is our hope that with their publication we may contribute to the spreading of the interest of actual and future researchers for the exciting world of dynamical systems.

Montserrat Corbera, Josep M. Cors and Enrique Ponce

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