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Dynamics and Mission Design Near Libration Points

Vol. I Fundamentals:
The Case of
Collinear Libration Points

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Preface

The present work is the final report of a study contract that was done for the European Space Agency and was finished in 1985. At that moment it was part of the mission analysis studies done by ESA for the SOHO mission. Unfortunately, none of the techniques developed in our work was used for the real mission, and the tools used for the determination of the nominal trajectory, the transfer from the Earth and the station keeping, were the same ones used, at the end of the seventies, for the ISEE-C mission, requiring a larger cost. Now the scenario has changed. Most of the missions to the Lagrange points are studied using the dynamical systems tools and ideas of our work. In general, they can be used when the basic model is a (perturbed) restricted three-body problem or some similar simple model which can be studied extensively. They have been shown to give a global and geometric picture of the problem, a clear understanding of the phase space and, at the same time, they provide a systematic approach that avoids the “trial and error” procedures, so widely used in the study of this kind of missions.

In this book the problem of station keeping is studied for orbits near libration points in the solar system. Main attention is devoted to orbits near halo ones in the Sun–Earth+Moon system. Taking as starting point the restricted three-body problem, the motion in the full solar system is considered as a perturbation of this simplified model. All the study is done with enough generality to allow an easy application to other primary–secondary systems as an easy extension of the analytic and numerical algorithms.

First, the families of halo orbits and the motion near them are studied. Then, the general equations of motion are stated and integrated to get the final nominal quasi-periodic orbit. Once the nominal orbit is available, an on/off station keeping method is developed, founded on geometrical considerations that use ideas of dynamical systems theory. The feasibility of radiation pressure and low thrust station keeping has been shown. It has been proved that the proposed on/off method of station keeping is very cheap, in terms of fuel consumption, and the numerical simulations can be done in a very fast way, when the nominal orbit and projection factors are available.

All the above items have been studied numerically and, as far as possible, analytically.

For the ESA original study, we produced a large amount of software to perform the analytic and numerical computations and to do simulations of the real behavior. This software is not included in the text, but all its main modules are described with detail.

The ESA report is reproduced textually with minor modifications: the detected typing or obvious mistakes have been corrected and some tables have been shortened. The layout of the (scanned) figures and tables has changed slightly, to accommodate to latex requirements.

The last page of this preface reproduces the cover page of the report for the European Space Agency showing, in particular, the original title of the study.

This is the first of a series of four reports done by persons of our research team for ESA. These four reports are now reprinted by World Scientific Pub. Co., to make available this information. Together they form a collection of works on *Dynamics and Mission Design Near Libration Points*.

All the data concerning performance of programs correspond to the time when the programs were developed, tested and executed. Some of them are now 17 years old. In this period the facilities available to us have improved by a factor ranging between 5×10^2 and 5×10^4 . The theoretical understanding, the algorithms and their implementation have also been largely improved, leading to a gain in speed, in some cases, by a factor up to 10^3 . It is not surprising that computations requiring 2 weeks of CPU time in 1983 take now 1 second.

Updates on the state of the art, both concerning theoretical and practical studies, can be found at the end of Volume IV of this collection. These updates refer to contributions done, in this domain, by our research team.

We are indebted to all the staff of World Scientific Pub. Co. for the cooperation and support in the preparation and publication of these Volumes, and to Prof. Rafael de la Llave for encouraging us in this task and his unvaluable help. Our deep acknowledgement to Dr. Walter Flury from the European Space Operations Center in Darmstadt for his kindness in writing the Foreword.

Barcelona, Fall 2000

On behalf of the authors
Carles Simó

STATION KEEPING OF LIBRATION POINT ORBITS FINAL REPORT

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EUROPEAN SPACE AGENCY. CONTRACT REPORT

The work described in this report is done under ESA contract.
Responsibility for the contents resides in the authors that prepared it

Barcelona, November, 1985

To the memory of
Dr. Ernest A. Roth
A Space Science Man

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